



Jet\_fitting\_3\_ref

Separate Build

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# Estimation of Local Differential Properties of Sampled Surfaces via Polynomial Fitting

## Reference Manual

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### Global Functions

The insert operator (*operator<<* ) is overloaded for the class *Monge\_form*.

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## DataKernel

### Definition

The concept `DataKernel` describes the set of requirements to be fulfilled by any class used to instantiate first template parameter of the class `Monge_via_jet_fitting<DataKernel,LocalKernel,KernelConverters,SvdTraits>`.

### Types

*DataKernel::FT*

The scalar type.

*DataKernel::Point\_3*

The point type.

*DataKernel::Vector\_3*

The vector type.

### Operations

Only constructors (from 3 scalars and copy constructors) and access methods to coordinates  $x()$ ,  $y()$ ,  $z()$  are needed.

### See Also

The *LocalKernel* concept.

## KernelConverters<DataKernel, LocalKernel>

### Definition

The concept `KernelConverters<DataKernel, LocalKernel>` describes the set of requirements to be fulfilled by any class used to instantiate third template parameter of the class `Monge_via_jet_fitting<DataKernel, LocalKernel, KernelConverters, SvdTraits>`. It enables conversions forth and back between number types, points and vectors of the `DataKernel` and `LocalKernel`.

### Operations

*LocalKernel::Point\_3*

*Kc.D2L\_converter( DataKernel::Point\_3 p)*

returns a *LocalKernel::Point\_3* which coordinates are those of *p* converted to the *LocalKernel::FT*.

The same function is also defined for the number types and the vector types.

*DataKernel::Point\_3*    *Kc.L2D\_converter( LocalKernel::Point\_3 p)*

returns a *DataKernel::Point\_3* which coordinates are those of *p* converted to the *DataKernel::FT*.

The same function is also defined for the number types and the vector types.

### See Also

The *LocalKernel* and the *DataKernel* concepts.

## LocalKernel

### Definition

The concept `LocalKernel` describes the set of requirements to be fulfilled by any class used to instantiate the second template parameter of the class `Monge_via_jet_fitting<DataKernel,LocalKernel,KernelConverters,SvdTraits>`.

This concept provides the geometric primitives used for the computations in the class `Monge_via_jet_fitting`.

### Requirements

In the class `Monge_via_jet_fitting` the scalar type, `LocalKernel::FT`, must be the same as that of the `SvdTraits` concept : `SvdTraits::FT`.

### Types

<code>LocalKernel:: FT</code>	The scalar type.
<code>LocalKernel:: Point_3</code>	The point type.
<code>LocalKernel:: Vector_3</code>	The vector type.
<code>LocalKernel:: LKMatrix</code>	For dimension 2 and 3 square matrices.
<code>LocalKernel:: Aff_transformation</code>	For 3d affine tranformation.

### Operations

The scalar type `LocalKernel::FT` must be a field type with a square root.  
`FT` `LK.Lsqr( FT x)`

Only constructors (from 3 scalars and copy constructors) and access methods to coordinates  $x()$ ,  $y()$ ,  $z()$  are needed for the point and vector types.

## MATRICES

### Definition

An instance of data type `LKMatrix` is a matrix of variables of number type `FT`.

### Types

`LKMatrix::iterator` : bidirectional iterator for accessing all components row-wise.

### Creation

`LKMatrix M(int n)`: creates an instance `M` of type `LKMatrix` of dimension  $n \times n$  initialized to the zero matrix.

## Operations

*LKMatrix*                      *LK.inverse( LKMatrix M, FT& D)*

returns the inverse matrix of M. More precisely, 1/D times the matrix returned is the inverse of M. Precondition: *determinant(M) != 0*. Precondition: M is square.

*int*                              *LK.sign\_of\_determinant( LKMatrix M)*

returns the sign of the determinant of M. Precondition: M is square.

## Affine Transformations

### Definition

The class *Aff\_transformation* represents three-dimensional affine transformations.

### Creation

*Aff\_transformation t( const FT &m00, const FT &m01, const FT &m02, const FT &m03, const FT &m10, const FT &m11, const FT &m12, const FT &m13, const FT &m20, const FT &m21, const FT &m22, const FT &m23);*

introduces a general affine transformation; the matrix  $m_{ij}$  for i and j from 0 to 2 defines the scaling and rotational part of the transformation, while the vector  $(m_{03}, m_{13}, m_{23})$  contains the translational part.

*Aff\_transformation t( const FT &m00, const FT &m01, const FT &m02, const FT &m10, const FT &m11, const FT &m12, const FT &m20, const FT &m21, const FT &m22);*

introduces a general affine transformation without translational part.

## Operations

*Aff\_transformation\_3 t.operator\*( s);* composes two affine transformations.

*Aff\_transformation\_3 t.inverse ();* gives the inverse transformation.

## Eigen Decomposition of a Symmetric Matrix

*void*                              *LK.eigen\_symmetric( const FT \*mat,*  
   *const int n,*  
   *FT \*eigen\_vectors,*  
   *FT \*eigen\_values)*

Computes eigenvalues and eigenvectors of a dimension n symmetric matrix. The matrix is given by the coefficients of its lower part row-wise. Eigenvalues are sorted in descending order, eigenvectors are sorted in accordance.

## See Also

The *DataKernel* and *SvdTraits* concepts.

## CGAL::Monge\_via\_jet\_fitting< DataKernel, LocalKernel, KernelConverters, SvdTraits>::Monge\_form

### Definition

The class *Monge\_form* stores the Monge representation, i.e., the Monge coordinate system and the coefficients of the Monge form in this system.

### Types

```
typedef typename DataKernel::FT
                        FT;
typedef typename DataKernel::Point_3
                        Point_3;
typedef typename DataKernel::Vector_3
                        Vector_3;
```

### Creation

*Monge\_form* *monge\_form*;                      default constructor.

### Access Functions

*Point\_3*                      *monge\_form.origin()*      Point on the fitted surface where differential quantities are computed.

The Monge basis is given by:

```
Vector_3                      monge_form.maximal_principal_direction()
Vector_3                      monge_form.minimal_principal_direction()
Vector_3                      monge_form.normal_direction()
```

The Monge coefficients are given by:

```
FT                              monge_form.principal_curvatures( size_t i )

                                 $i = 0$  for the maximum and  $i = 1$  for the minimum.

FT                              monge_form.third_order_coefficients( size_t i )

                                 $0 \leq i \leq 3$ 

FT                              monge_form.fourth_order_coefficients( size_t i )

                                 $0 \leq i \leq 4$ 
```

## Operations

*void* *monge\_form.comply\_wrt\_given\_normal( const Vector\_3 given\_normal)*

change principal basis and Monge coefficients so that the *given\_normal* and the Monge normal make an acute angle.  
If *given\_normal.monge\_normal* < 0 then change the orientation: if  $z = g(x,y)$  in the basis (d1,d2,n) then in the basis (d2,d1,-n)  $z = h(x,y) = -g(y,x)$ .

The usual output operator (*operator<<*) is overloaded for *Monge\_form*, it gives the Monge coordinate system (the origin and an orthonormal basis) and the coefficients of the Monge form in this system.

## See Also

*Monge\_via\_jet\_fitting*

## CGAL::Monge\_via\_jet\_fitting<DataKernel, LocalKernel, KernelConverters, SvdTraits>

### Definition

The class *Monge\_via\_jet\_fitting*<*DataKernel*, *LocalKernel*, *KernelConverters*, *SvdTraits*> is designed to perform the estimation of the local differential quantities at a given point. The point range is given by a pair of input iterators, and it is assumed that the point where the calculation is carried out is the point that the begin iterator refers to. The results are stored in an instance of the nested class *Monge\_form*, the particular information returned depending on the degrees specified for the polynomial fitting and for the Monge form.

### Parameters

The class *Monge\_via\_jet\_fitting*<*DataKernel*, *LocalKernel*, *KernelConverters*, *SvdTraits*> has four template parameters. Parameter *DataKernel* provides the geometric classes and tools corresponding to the input points, and also members of the *Monge\_form* class. Parameter *LocalKernel* provides the geometric classes and tools required by local computations. Parameter *KernelConverters* enables conversions of kernel geometric classes. Parameter *SvdTraits* features the linear algebra algorithm required by the fitting method.

### Types

```
typedef DataKernel      Data_kernel;
typedef LocalKernel     Local_kernel;
typedef typename Local_kernel::FT

                        FT;
typedef typename Local_kernel::Vector_3

                        Vector_3;
typedef typename DataKernel::Vector_3

                        DVector_3;
```

*Monge\_via\_jet\_fitting*<*DataKernel*, *LocalKernel*, *KernelConverters*, *SvdTraits*>:: *Monge\_form*

see its specific the section.

### Creation

*Monge\_via\_jet\_fitting*<*DataKernel*, *LocalKernel*, *KernelConverters*, *SvdTraits*> *monge\_fitting*;

default constructor

## Operations

```
template <class InputIterator>
Monge_form      monge_fitting( InputIterator begin, InputIterator end, size_t d, size_t d')
```

This operator performs all the computations. The  $N$  input points are given by the *InputIterator* parameters which value-type are *Data\_kernel::Point\_3*,  $d$  is the degree of the fitted polynomial,  $d'$  is the degree of the expected Monge coefficients.

*Precondition:*  $N \geq N_d := (d + 1)(d + 2)/2$ ,  $1 \leq d' \leq \min(d, 4)$ .

```
template <class InputIterator>
Monge_form      monge_fitting.operator()( InputIterator begin,
                                           InputIterator end,
                                           size_t d,
                                           size_t d',
                                           DVector_3 vx,
                                           DVector_3 vy,
                                           DVector_3 vz)
```

This operator performs the same computations as the former. The difference is that the coordinate system in which the fitting is performed is given by the orthonormal basis (vx, vy, vz).

```
FT      monge_fitting.condition_number()
```

condition number of the linear fitting system.

```
std::pair<FT, Vector_3>
```

```
monge_fitting.pca_basis( size_t i)
```

pca eigenvalues and eigenvectors, the *pca\_basis* has always 3 such pairs. *Precondition* :  $i$  ranges from 0 to 2.

## See Also

*Monge\_form*

## SvdTraits

### Definition

The concept `SvdTraits` describes the set of requirements to be fulfilled by any class used to instantiate the fourth template parameter of the class `Monge_via_jet_fitting<DataKernel,LocalKernel,KernelConverters,SvdTraits>`.

It describes the linear algebra types and algorithms needed by the class `Monge_via_jet_fitting`.

### Requirements

The scalar type, `SvdTraits::FT`, must be the same as that of the `LocalKernel` concept : `LocalKernel::FT`.

### Types

<code>SvdTraits:: FT</code>	The scalar type.
<code>SvdTraits:: Vector</code>	The vector type.
<code>SvdTraits:: Matrix</code>	The matrix type.

### Operations

`SvdTraits vector( size_t n);` initialize all the elements of the vector to zero.

The type `Vector` has the access methods

<code>size_t</code>	<code>vector.size()</code>	
<code>FT</code>	<code>vector( size_t i)</code>	return the $i^{th}$ entry, $i$ from 0 to <code>size()</code> - 1.
<code>void</code>	<code>vector.set( size_t i, const FT value)</code>	set the $i^{th}$ entry to <code>value</code> .

The type `Matrix` has the access methods

`SvdTraits matrix( size_t n1, size_t n2);` initialize all the entries of the matrix to zero.

<code>size_t</code>	<code>matrix.number_of_rows()</code>	
<code>size_t</code>	<code>matrix.number_of_columns()</code>	
<code>FT</code>	<code>matrix( size_t i, size_t j)</code>	return the entry at row $i$ and column $j$ , $i$ from 0 to <code>number_of_rows</code> - 1, $j$ from 0 to <code>number_of_columns</code> - 1.
<code>void</code>	<code>matrix.set( size_t i, size_t j, const FT value)</code>	set the entry at row $i$ and column $j$ to <code>value</code> .

The concept `SvdTraits` has a linear solver using a singular value decomposition algorithm.

*FT*

*traits.solve( Matrix& M, Vector& B)*

Solves the system  $MX = B$  (in the least square sense if  $M$  is not square) using a singular value decomposition and returns the condition number of  $M$ . The solution is stored in  $B$ .

**See Also**

*LocalKernel*

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