LSA: Optimization subroutines

1. Description of subroutines

In this section we describe easy-to-use subroutines PLISU, PLISS, PLIPU, PLIPS, PNETU, PNETS, PNEDU, PNEDS, PNECU, PNECS, PSEDU, PSEDS, PSECU, PSECS, PSENU, PGADU, PGADS, PGACU, PGACS, PMAXU, PSUMU, PEQNU, PEQLU, which can be called from the user's program. In the description of formal parameters we introduce a type of the argument denoted by two letters. The first letter is either I for integer arguments or R for double-precision real arguments. The second letter specifies whether the argument must have a value defined on the entry to the subroutine (I), whether it is a value which will be returned (O), or both (U), or whether it is an auxiliary value (A). Besides the formal parameters, we use a COMMON /STAT/ block containing statistical information. This block, used in each subroutine, has the form

```
COMMON /STAT/ NRES,NDEC,NIN,NIT,NFV,NFG,NFH
```

whose elements have the following meanings:

Element	Type	Significance
NRES	IO	Number of restarts.
NDEC	IO	Number of matrix decompositions.
NIN	IO	Number of inner iterations (for solving linear systems).
NIT	IO	Number of iterations.
NFV	IO	Number of function evaluations.
NFG	IO	Number of gradient evaluations.
NFH	IO	Number of Hessian evaluations.

Easy-to-use subroutines are called by the following statements:

```
CALL PLISU(NF,X,IPAR,RPAR,F,GMAX,IPRNT,ITERM)
CALL PLISS(NF,X,IX,XL,XU,IPAR,RPAR,F,GMAX,IPRNT,ITERM)
CALL PLIPU(NF,X,IPAR,RPAR,F,GMAX,IPRNT,ITERM)
CALL PLIPS(NF,X,IX,XL,XU,IPAR,RPAR,F,GMAX,IPRNT,ITERM)
CALL PNETU(NF,X,IPAR,RPAR,F,GMAX,IHES,IPRNT,ITERM)
CALL PNETS(NF,X,IX,XL,XU,IPAR,RPAR,F,GMAX,IHES,IPRNT,ITERM)
CALL PNEDU(NF,MH,X,IH,JH,IPAR,RPAR,F,GMAX,ISPAS,IPRNT,ITERM)
CALL PNEDS(NF,MH,X,IX,XL,XU,IH,JH,IPAR,RPAR,F,GMAX,ISPAS,IPRNT,ITERM)
CALL PNECU(NF,MH,X,IH,JH,IPAR,RPAR,F,GMAX,ISPAS,IPRNT,ITERM)
CALL PNECS(NF,MH,X,IX,XL,XU,IH,JH,IPAR,RPAR,F,GMAX,ISPAS,IPRNT,ITERM)
CALL PSEDU(NF,NA,MA,X,AF,IAG,JAG,IPAR,RPAR,F,GMAX,ISPAS,IPRNT,ITERM)
CALL PSEDS(NF,NA,MA,X,IX,XL,XU,AF,IAG,JAG,IPAR,RPAR,F,GMAX,ISPAS,IPRNT,ITERM)
CALL PSECU(NF,NA,MA,X,AF,IAG,JAG,IPAR,RPAR,F,GMAX,ISPAS,IPRNT,ITERM)
CALL PSECS(NF,NA,MA,X,IX,XL,XU,AF,IAG,JAG,IPAR,RPAR,F,GMAX,ISPAS,IPRNT,ITERM)
CALL PSENU(NF,NA,MA,X,AF,IAG,JAG,IPAR,RPAR,F,GMAX,ISPAS,IPRNT,ITERM)
CALL PGADU(NF,NA,MA,X,AF,IAG,JAG,IPAR,RPAR,F,GMAX,IDER,ISPAS,IPRNT,ITERM)
CALL PGADS(NF,NA,MA,X,IX,XL,XU,AF,IAG,JAG,IPAR,RPAR,F,GMAX,IDER,ISPAS,IPRNT,ITERM)
CALL PGACU(NF,NA,MA,X,AF,IAG,JAG,IPAR,RPAR,F,GMAX,IDER,ISPAS,IPRNT,ITERM)
CALL PGACS(NF,NA,MA,X,IX,XL,XU,AF,IAG,JAG,IPAR,RPAR,F,GMAX,IDER,ISPAS,IPRNT,ITERM)
CALL PMAXU(NF,NA,MA,X,AF,IAG,JAG,IPAR,RPAR,F,GMAX,IEXT,ISPAS,IPRNT,ITERM)
```

```
CALL PSUMU(NF,NA,MA,X,AF,IAG,JAG,IPAR,RPAR,F,GMAX,ISPAS,IPRNT,ITERM)
CALL PEQNU(N,MA,X,AF,IAG,JAG,IPAR,RPAR,F,GMAX,IDER,ISPAS,IPRNT,ITERM)
CALL PEQLU(N,MA,X,AF,IAG,JAG,IPAR,RPAR,F,GMAX,IDER,ISPAS,IPRNT,ITERM)
```

Their arguments have the following meanings:

Argument	Type	Significance
NF	II	Number of variables of the objective function.
NA	II	Number of partial functions.
N	II	Number of nonlinear equations and unknowns.
MH	II	Number of nonzero elements in the upper part of the Hessian matrix. This parameter is used as input only if ISPAS = 1 (it defines dimensions of arrays IH and JH in this case).
MA	II	Number of nonzero elements in the Jacobian matrix. This parameter is used as input only if ISPAS = 1 (it defines dimensions of arrays IAG and JAG in this case).
X(NF)	RU	On input, vector with the initial estimate to the solution. On output, the approximation to the minimum.
IX(NF)	II	Vector containing the box constraint types (significant only if box constraints are considered): IX(I) = 0: the variable X(I) is unbounded, IX(I) = 1: the lower bound X(I) ≥ XL(I), IX(I) = 2: the upper bound X(I) ≤ XU(I), IX(I) = 3: the two-side bound XL(I) ≤ X(I) ≤ XU(I), IX(I) = 5: the variable X(I) is fixed (given by its initial estimate).
XL(NF)	RI	Vector with lower bounds for variables (significant only if box constraints are considered).
XU(NF)	RI	Vector with upper bounds for variables (significant only if box constraints are considered).
IH(NIH)	IU	Row indices of the nonzero elements in the upper part of the Hessian matrix (NIH = NF + MH) if ISPAS = 1. Indices in array JH of the diagonal elements of the Hessian matrix (NIH = NF + 1) if ISPAS = 2.
JH(NJH)	IU	Column indices of the nonzero elements in the upper part of the Hessian matrix (NJH = NF + MH if ISPAS = 1 or NJH = MH if ISPAS = 2).
AF(NA)	RO	Vector which contains values of partial functions.
IAG(NIAG)	IU	Row indices of nonzero elements of the Jacobian matrix (NIAG = NA + MA) if ISPAS = 1. Indices in array JAG of the first elements in the rows of the Jacobian matrix (NIAG = NA + 1) if ISPAS = 2.
JAG(NJAG)	IU	Column indices of nonzero elements of the Jacobian matrix (NJAG = NA + MA if ISPAS = 1 or NJAG = MA if ISPAS = 2).
IPAR(7)	IU	Integer parameters (see Table 1).
RPAR(9)	RU	Real parameters (see Table 1).
F	RO	Value of the objective function at the solution X.
GMAX	RO	Maximum absolute value (l_∞ norm) of the gradient of the Lagrangian function.
IEXT	II	The type of minimax: IEXT < 0: minimization of the maximum positive value, IEXT = 0: minimization of the maximum absolute value, IEXT > 0: minimization of the maximum negative value.

`IDER`	`II`	Degree of analytically computed derivatives (0 or 1).
`IHES`	`II`	The way for computing the product of the Hessian matrix and a vector (0 or 1): `IHES` = 0: the product is computed by using the gradient differences, `IHES` = 1: the product is computed by using the user supplied subroutine `HVEC`.
`ISPAS`	`II`	Sparse structure of the Hessian or Jacobian matrix: `ISPAS` = 1: the coordinate form is used, `ISPAS` = 2: the standard row compressed format is used.
`IPRNT`	`II`	Print specification: `IPRNT` = 0: print is suppressed, `IPRNT` = 1: basic print of final results, `IPRNT` = −1: extended print of final results, `IPRNT` = 2: basic print of intermediate and final results, `IPRNT` = −2: extended print of intermediate and final results.
`ITERM`	`IO`	Variable that indicates the cause of termination: `ITERM` = 1: if ‖`X` − `XO`‖ was less than or equal to `TOLX` in two subsequent iterations (`XO` is the vector of variables in the previous iteration), `ITERM` = 2: if |`F` − `FO`| was less than or equal to `TOLF` in two subsequent iterations (`FO` is the function value in the previous iteration), `ITERM` = 3: if `F` is less than or equal to `TOLB`, `ITERM` = 4: if `GMAX` is less than or equal to `TOLG`, `ITERM` = 6: if termination criterion was not satisfied, but the solution is probably acceptable, `ITERM` = 11: if `NIT` exceeded `MIT`, `ITERM` = 12: if `NFV` exceeded `MFV`, `ITERM` = 13: if `NFG` exceeded `MFG`, `ITERM` < 0: if the method failed.

Integer and real parameters `IPAR` and `RPAR` need not be defined by the user who is not familiar with details of optimization methods. If we assign zeroes to these parameters, the suitable default values are assumed. An advanced user can change these default values by his knowledge-based options. The individual parameters, presented in Table 1, have the following meanings:

Parameter	PLIS	PLIP	PNET	PNED	PNEC	PSED	PSEC	PSEN	PGAD	PGAC	PMAX	PSUM	PEQN	PEQL
IPAR(1)	MIT	MIT	MIT	MIT	MIT	MIT	MIT	MIT	MIT	MIT	MIT	MIT	MIT	MIT
IPAR(2)	MFV	MFV	MFV	MFV	MFV	MFV	MFV	MFV	MFV	MFV	MFV	MFV	MFV	MFV
IPAR(3)	-	-	MFG	MFG	MFG	MFG	MFG	-	MFG	MFG	MFG	MFG	-	-
IPAR(4)	IEST	IEST	IEST	IEST	IEST	IEST	IEST	IEST	MEC	MEC	IEST	IEST	-	-
IPAR(5)	-	MET	MOS1	MOS	MOS1	MET	MET	-	MOS	MOS1	MED	MED	MOS1	MOS1
IPAR(6)	-	-	MOS2	-	MOS2	-	MOS2	MB	-	MOS2	-	-	MOS2	MOS2
IPAR(7)	MF	MF	MF	IFIL	IFIL	IFIL	IFIL	IFIL	IFIL	IFIL	IFIL	IFIL	-	MF
RPAR(1)	XMAX	XMAX	XMAX	XMAX	XMAX	XMAX	XMAX	XMAX	XMAX	XMAX	XMAX	XMAX	XMAX	XMAX
RPAR(2)	TOLX	TOLX	TOLX	TOLX	TOLX	TOLX	TOLX	TOLX	TOLX	TOLX	TOLX	TOLX	TOLX	TOLX
RPAR(3)	TOLF	TOLF	TOLF	TOLF	TOLF	TOLF	TOLF	TOLF	TOLF	TOLF	TOLF	TOLF	TOLF	TOLF
RPAR(4)	TOLB	TOLB	TOLB	TOLB	TOLB	TOLB	TOLB	TOLB	TOLB	TOLB	TOLB	TOLB	TOLB	TOLB
RPAR(5)	TOLG	TOLG	TOLG	TOLG	TOLG	TOLG	TOLG	TOLG	TOLG	TOLG	TOLG	TOLG	TOLG	TOLG
RPAR(6)	FMIN	FMIN	FMIN	FMIN	FMIN	FMIN	FMIN	FMIN	FMIN	FMIN	FMIN	FMIN	-	-
RPAR(7)	-	-	-	XDEL	XDEL	-	-	-	XDEL	XDEL	-	XDEL	-	-
RPAR(8)	-	-	-	-	-	-	-	ETA3	ETA	ETA	ETA4	-	ETA2	ETA2
RPAR(9)	-	-	-	-	-	-	-	ETA5	-	-	ETA5	ETA5	-	-

Table 1. Integer and real parameters

Argument	Type	Significance
MIT	II	Maximum number of iterations; the choice MIT = 0 causes that the default value (see Table 2) will be taken.
MFV	II	Maximum number of function evaluations; the choice MFV = 0 causes that the default value (see Table 2) will be taken.
MFG	II	Maximum number of gradient evaluations; the choice MFG = 0 causes that the default value (see Table 2) will be taken.
IEST	II	Estimation of the minimum function value for the line search: IEST = 0: estimation is not used, IEST = 1: lower bound FMIN is used as an estimation for the minimum function value.
MEC	II	Variable that determines the method of a second order correction: MEC = 1: correction by the Marwil sparse variable metric update, MEC = 2: correction by differences of gradients (discrete Newton correction), MEC = 3: correction by the Griewank–Toint partitioned variable metric update (symmetric rank-one). The choice MEC = 0 causes that the default value MEC = 2 will be taken.
MED	II	Variable that specifies the method used: MED = 1: partitioned variable metric method, MED = 2: safeguarded discrete Newton method. The choice MED = 0 causes that the default value MED = 1 will be taken.
MET	II	In PLIP: Variable that specifies the limited-memory method: MET = 1: rank-one method, MET = 2: rank-two method. The choice MET = 0 causes that the default value MET = 2 will be taken. In PSED,PSEC: Variable that specifies the variable metric update:

		MET = 1: safeguarded BFGS method, MET = 2: combination of the BFGS and the symmetric rank-one method, MET = 3: discrete Newton method. The choice MET = 0 causes that the default value MET = 2 will be taken.
MOS	II	Method for computing the trust-region step: MOS = 1: double dog-leg method of Dennis and Mei, MOS = 2: method of More and Sorensen for obtaining optimum locally constrained step. The choice MOS = 0 causes that the default value MOS = 2 will be taken.
MOS1	II	In PNET: Choice of restarts after constraint change: MOS1 = 1: restarts are suppressed, MOS1 = 2: restarts with steepest descent directions are used. The choice MOS1 = 0 causes that the default value MOS1 = 1 will be taken. In PNEC, PGAC: Method for computing trust-region step: MOS1 = 1: Steihaug–Toint conjugate gradient method, MOS1 = 2: shifted Steihaug–Toint method with five Lanczos steps, MOS1 $>$ 2: shifted Steihaug–Toint method with MOS1 Lanczos steps. The choice MOS1 = 0 causes that the default value MOS1 = 2 will be taken. In PEQN,PEQL: Variable that specifies the smoothing strategy for the CGS method: MOS1 = 1: smoothing is not used, MOS1 = 2: single smoothing strategy is used, MOS1 = 3: double smoothing strategy is used. The choice MOS1 = 0 causes that the default value MOS1 = 3 will be taken.
MOS2	II	Choice of preconditioning strategy: MOS2 = 1: preconditioning is not used, MOS2 = 2: preconditioning by the incomplete Gill-Murray decomposition (in PNET,PNEC,PSEC,PGAC) or incomplete LU decomposition in PEQN,PEQL), MOS2 = 3: preconditioning as in case MOS2; the preliminary solution of the preconditioned system is accepted if it satisfies the termination criteria. The choice MOS2 = 0 causes that the default values MOS2 = 1 in PNET, MOS2 = 2 in PNEC,PSEC,PGAC or MOS2 = 3 in PEQN,PEQL will be taken.
MB	II	Dimension of a bundle used in the line search; the choice MB = 0 causes that the default value MB = 20 will be taken.
MF	II	The number of limited-memory variable metric updates in every iteration; the choice MF = 0 causes that the default values MF = 10 in PLIS,PLIP,PNET or MF = 6 in PEQL will be taken.
IFIL	II	Variable that specifies a relative size of the space reserved for fill-in; the choice IFIL = 0 causes that the default value IFIL = 1 will be taken.
XMAX	RI	Maximum stepsize; the choice XMAX = 0 causes that the default value (see Table 2) will be taken.
TOLX	RI	Tolerance for the change of the coordinate vector X; the choice TOLX = 0 causes that the default value (see Table 2) will be taken.
TOLF	RI	Tolerance for the change of function values; the choice TOLF = 0 causes that the default value (see Table 2) will be taken.
TOLB	RI	Minimum acceptable function value; the choice TOLB = 0 causes that the default value (see Table 2) will be taken.

TOLG	RI	Tolerance for the Lagrangian function gradient; the choice TOLG = 0 causes that the default value (see Table 2) will be taken.
FMIN	RI	Lower bound for the minimum function value. This value is not used if IEST=0.
XDEL	RI	Trust region step-size; the choice XDEL = 0 causes that a suitable default value will be computed.
ETA	RI	Parameter for switch between the Gauss-Newton method and variable metric correction; the choice ETA = 0 causes that the default value ETA = $1.5 \cdot 10^{-4}$ will be taken.
ETA2	RI	Damping parameter for an incomplete LU preconditioner; the choice ETA2 = 0 causes that the default value ETA2 = 0 will be taken.
ETA3	RI	Correction parameter; the choice ETA3 = 0 causes that the default value ETA3 = 10^{-12} will be taken.
ETA4	RI	Coefficient for the barrier parameter decrease; the choice ETA4 = 0 causes that the default value ETA4 = 0.85 will be taken.
ETA5	RI	In PSEN: Parameter for subgradient locality measure; the choice ETA5 = 0 causes that the default value ETA5 = 10^{-12} will be taken. In PMAX,PSUM: Minimum permitted value of the barrier parameter; the choice ETA5 = 0 causes that the default value ETA5 = 10^{-10} in PMAX and ETA5 = 10^{-8} in PSUM will be taken.

Value	PLIS PLIP	PNET	PNED PNEC	PSED PSEC	PSEN	PGAD PGAC	PMAX	PSUM	PEQL PEQN
MIT	9000	5000	5000	9000	20000	5000	10000	10000	1000
MFV	9000	5000	5000	9000	20000	5000	10000	10000	1000
MFG	9000	30000	10000	9000	20000	10000	20000	20000	10000
XMAX	10^{16}	10^{16}	10^{16}	10^{16}	10^{16}	10^{16}	10^{16}	10^{16}	10^{16}
TOLX	10^{-16}	10^{-16}	10^{-16}	10^{-16}	10^{-16}	10^{-16}	10^{-16}	10^{-16}	10^{-16}
TOLF	10^{-14}	10^{-14}	10^{-14}	10^{-14}	10^{-12}	10^{-14}	10^{-14}	10^{-12}	10^{-16}
TOLB	10^{-16}	10^{-16}	10^{-16}	10^{-16}	10^{-12}	10^{-16}	10^{-16}	10^{-12}	10^{-16}
TOLG	10^{-6}	10^{-6}	10^{-6}	10^{-6}	10^{-8}	10^{-6}	10^{-6}	10^{-6}	10^{-6}

Table 2. Default values

The subroutines PLISU,PLISS,PLIPU,PLIPS,PNETU,PNETS,PNEDU,PNEDS,PNECU,PNECS require the user supplied subroutines OBJ,DOBJ that define the objective function and its gradient and have the form

```
SUBROUTINE  OBJ(NF,X,F)
SUBROUTINE DOBJ(NF,X,G)
```

The subroutines PNETU,PNETS require the user supplied subroutine HVEC that defines the product of the Hessian matrix and a vector and have the form

```
SUBROUTINE HVEC(NF,X,D,HD)
```

If IHES = 0, the subroutine HVEC can be empty. The subroutines PSEDU,PSEDS,PSECU,PSECS,PSENU,PGADU, PGADS,PGACU,PGACS,PMAXU,PSUMU,PEQNU,PEQLU require the user supplied subroutines FUN,DFUN that define particular functions and their gradients and have the form

```
SUBROUTINE  FUN(NF,KA,X,FA)
SUBROUTINE DFUN(NF,KA,X,GA)
```

If IDER = 0, the subroutine DFUN can be empty. The arguments of the user supplied subroutines have the following meanings:

Argument	Type	Significance
NF	II	Number of variables, equations and unknowns.
KA	II	Index of the partial function.
X(NF)	RI	An estimate to the solution.
F	RO	Value of the objective function at the point X.
FA	RO	Value of the KA-th partial function at the point X.
G(NF)	RO	Gradient of the objective function at the point X.
GA(NF)	RO	Gradient of the KA-th smooth partial function (or an arbitrary subgradient of the KA-th nonsmooth partial function) at the point X.
D(NF)	RI	Input vector.
HD(NF)	RO	Product of the Hessian matrix and the vector D.

2. Verification of subroutines

In this section we report the results obtained by using test programs TLISU, TLISS, TLIPU, TLIPS, TNETU, TNETS, TNEDU, TNEDS, TNECU, TNECS, TSEDU, TSEDS, TSECU, TSECS, TSENU, TGADU, TGADS, TGACU, TGACS, TMAXU, TSUMU, TEQNU, TEQLU, which serve for demonstration, verification and testing of subroutines PLISU, PLISS, PLIPU, PLIPS, PNETU, PNETS, PNEDU, PNEDS, PNECU, PNECS, PSEDU, PSEDS, PSECU, PSECS, PSENU, PGADU, PGADS, PGACU, PGACS, PMAXU, PSUMU, PEQNU, PEQLU. These results are listed in the following tables (rows corresponding to individual test problems contain the number of iterations NIT, the number of function evaluations NFV, the number of gradient evaluations NFG, the final value of the objective function F, the value of the termination criterion G, and the cause of termination ITERM). The last row of every table contains the total number of NIT, NFV, NFG, and, moreover, for programs TNECU, TNECS, TSECU, TSECS, TGACU, TGACS, TEQNU, TEQLU also the total number of conjugate gradient iterations NCG. The total computational time in seconds is included. All computations reported were performed on a Pentium PC computer, under the Windows XP system using the Digital Visual Fortran (Version 6) compiler, in double-precision arithmetic.

The above test programs are based on test collections TEST14, TEST15, TEST18 described in [1], which can be downloaded from www.cs.cas.cz/~luksan/test.html. The box constraints are assumed in the form $-1 \le x_i \le 1$, $1 \le i \le n$. Subroutines for nonlinear equations were tested by problems with 3000 equations and unknowns. Subroutines for nonsmooth unconstrained optimization were tested by problems with 200 variables. The remaining subroutines were tested by problems with 1000 variables.

Problem	NIT	NFV	NFG	F	G	ITERM
1	4988	5554	5554	0.963780013E−14	0.891E−06	4
2	425	454	454	14.9944763	0.773E−05	2
3	74	78	78	0.655101686E−09	0.539E−06	4
4	103	112	112	269.499543	0.899E−06	4
5	24	26	26	0.130639280E−11	0.671E−06	4
6	30	31	31	0.216102227E−10	0.946E−06	4
7	38	43	43	335.137433	0.730E−06	4
8	29	33	33	761774.954	0.432E−03	2
9	13	16	16	316.436141	0.369E−06	4
10	1540	1582	1582	−124.630000	0.124E−04	2
11	114	138	138	10.7765879	0.380E−06	4
12	248	267	267	982.273617	0.123E−04	2
13	7	8	8	0.165734137E−12	0.453E−06	4
14	10	12	12	0.128729169E−08	0.916E−06	4
15	2830	2929	2929	1.92401599	0.936E−06	4
16	196	210	210	−427.404476	0.991E−05	2
17	1007	1032	1032	−0.379921091E−01	0.876E−06	4
18	1449	1474	1474	−0.245741193E−01	0.862E−06	4
19	1393	1431	1431	59.5986241	0.259E−05	2
20	2129	2191	2191	−1.00013520	0.908E−06	4
21	2120	2169	2169	2.13866377	0.927E−06	4
22	1305	1346	1346	1.00000000	0.982E−06	4
Σ	20072	21136	21136		TIME = 10.81	

Table 3. Results obtained by program TLISU

Problem	NIT	NFV	NFG	F	G	ITERM
1	5055	5595	5595	0.00000000	0.000E+00	3
2	2016	2289	2289	3926.45961	0.304E−04	2
3	95	106	106	0.217616100E-12	0.780E−06	4
4	58	64	64	269.522686	0.124E−05	2
5	24	26	26	0.130639280E-11	0.671E−06	4
6	30	31	31	0.216102227E-10	0.946E−06	4
7	31	35	35	337.722479	0.776E−06	4
8	50	58	58	761925.725	0.257E−03	2
9	504	506	506	428.056916	0.940E−07	4
10	1152	1211	1211	−82.0207503	0.176E−04	2
11	13	23	23	96517.2947	0.126E−08	4
12	79	88	88	4994.21410	0.325E−06	4
13	7	8	8	0.165734137E-12	0.453E−06	4
14	10	12	12	0.128729169E-08	0.916E−06	4
15	2830	2929	2929	1.92401599	0.936E−06	4
16	176	184	184	−427.391653	0.348E−04	2
17	1007	1032	1032	−0.379921091E-01	0.876E−06	4
18	1449	1474	1474	−0.245741193E-01	0.862E−06	4
19	1150	1183	1183	1654.94525	0.908E−05	2
20	2211	2274	2274	−1.00013520	0.886E−06	4
21	1280	1303	1303	2.41354873	0.997E−06	4
22	1562	1598	1598	1.00000000	0.786E−06	4
Σ	20789	22029	22029		TIME = 13.70	

Table 4. Results obtained by program TLISS

Problem	NIT	NFV	NFG	F	G	ITERM
1	5383	5417	5417	0.601022658E−13	0.599E−06	4
2	530	557	557	3.57276719	0.124E−05	2
3	125	128	128	0.338270284E−12	0.518E−06	4
4	109	114	114	269.499543	0.669E−06	4
5	26	27	27	0.710072396E−11	0.951E−06	4
6	35	36	36	0.142942272E−10	0.737E−06	4
7	36	41	41	336.937181	0.956E−06	4
8	33	36	36	761774.954	0.192E−02	2
9	15	18	18	316.436141	0.264E−06	4
10	2003	2030	2030	−124.950000	0.116E−04	2
11	157	175	175	10.7765879	0.299E−06	4
12	337	350	350	982.273617	0.145E−04	2
13	9	10	10	0.230414406E−14	0.642E−07	4
14	8	10	10	0.128834241E−08	0.977E−06	4
15	1226	1256	1256	1.92401599	0.970E−06	4
16	237	246	246	−427.404476	0.501E−04	2
17	598	604	604	−0.379921091E−01	0.908E−06	4
18	989	998	998	−0.245741193E−01	0.975E−06	4
19	1261	1272	1272	59.5986241	0.410E−05	2
20	2045	2058	2058	−1.00013520	0.911E−06	4
21	2175	2196	2196	2.13866377	0.996E−06	4
22	1261	1292	1292	1.00000000	0.927E−06	4
Σ	18598	18871	18871		TIME = 10.82	

Table 5. Results obtained by program TLIPU

Problem	NIT	NFV	NFG	F	G	ITERM
1	5263	5321	5321	0.530131995E−13	0.370E−05	2
2	2293	2447	2447	3930.43962	0.251E−04	2
3	127	132	132	0.210550150E−12	0.437E−06	4
4	70	72	72	269.522686	0.794E−06	4
5	26	27	27	0.710072396E−11	0.951E−06	4
6	35	36	36	0.142942272E−10	0.737E−06	4
7	37	43	43	336.937181	0.133E−05	2
8	59	65	65	761925.725	0.399E−03	2
9	508	510	510	428.056916	0.776E−06	4
10	1253	1277	1277	−82.5400568	0.120E−04	2
11	13	19	19	96517.2947	0.150E−04	2
12	95	102	102	4994.21410	0.790E−04	2
13	9	10	10	0.230414406E−14	0.642E−07	4
14	8	10	10	0.128834241E−08	0.977E−06	4
15	1226	1256	1256	1.92401599	0.970E−06	4
16	227	228	228	−427.391653	0.952E−05	2
17	598	604	604	−0.379921091E−01	0.908E−06	4
18	989	998	998	−0.245741193E−01	0.975E−06	4
19	1367	1383	1383	1654.94525	0.105E−04	2
20	2274	2303	2303	−1.00013520	0.798E−06	4
21	1196	1211	1211	2.41354873	0.975E−06	4
22	1361	1381	1381	1.00000000	0.962E−06	4
Σ	19034	19435	19435		TIME = 11.39	

Table 6. Results obtained by program TLIPS

Problem	NIT	NFV	NFG	F	G	ITERM
1	1481	1656	26037	0.117631766E−15	0.354E−06	4
2	132	387	7945	0.153382199E−15	0.988E−08	4
3	19	20	110	0.421204156E−09	0.353E−06	4
4	19	20	230	269.499543	0.779E−07	4
5	12	13	49	0.465606821E−11	0.364E−06	4
6	13	14	76	0.366783327E−11	0.404E−06	4
7	9	10	37	336.937181	0.248E−06	4
8	11	12	58	761774.954	0.155E−07	4
9	7	11	28	316.436141	0.158E−07	4
10	70	143	2657	−129.170000	0.182E−07	4
11	33	45	181	10.7765879	0.414E−07	4
12	23	30	457	982.273617	0.591E−08	4
13	7	8	16	0.533593908E−15	0.327E−07	4
14	1	2	1005	0.120245125E−08	0.879E−07	4
15	14	15	4033	1.92401599	0.468E−07	4
16	13	17	295	−427.404476	0.800E−08	4
17	4	5	810	−0.379921091E−01	0.537E−06	4
18	4	5	1146	−0.245741193E−01	0.425E−06	4
19	10	11	1986	59.5986241	0.423E−06	4
20	18	39	3051	−1.00013520	0.712E−07	4
21	7	8	4901	2.13866377	0.120E−08	4
22	55	145	4760	1.00000000	0.206E−08	4
Σ	1962	2616	59868		TIME = 7.89	

Table 7. Results obtained by program TNETU

Problem	NIT	NFV	NFG	F	G	ITERM
1	1611	1793	28524	0.00000000	0.000E+00	3
2	259	259	4418	3930.43956	0.230E−07	4
3	17	18	98	0.158634811E−08	0.954E−06	4
4	12	13	105	269.522686	0.103E−07	4
5	12	13	49	0.465606821E−11	0.364E−06	4
6	13	14	76	0.366783327E−11	0.404E−06	4
7	9	10	37	336.937181	0.248E−06	4
8	40	41	248	761925.725	0.281E−06	4
9	553	555	2056	428.056916	0.850E−07	4
10	112	137	2109	−84.1426617	0.732E−06	4
11	7	8	17	96517.2947	0.112E−11	4
12	133	136	2689	4994.21410	0.180E−06	4
13	7	8	16	0.533593908E−15	0.327E−07	4
14	1	2	1005	0.120245125E−08	0.879E−07	4
15	14	15	4033	1.92401599	0.468E−07	4
16	12	13	294	−427.391653	0.594E−06	4
17	4	5	810	−0.379921091E−01	0.537E−06	4
18	4	5	1146	−0.245741193E−01	0.425E−06	4
19	8	9	1902	1654.94525	0.690E−07	4
20	16	25	3254	−1.00013520	0.836E−08	4
21	4	5	1211	2.41354873	0.135E−06	4
22	52	137	4843	1.00000000	0.657E−06	4
Σ	2900	3221	58940		TIME = 9.98	

Table 8. Results obtained by program TNETS

Problem	NIT	NFV	NFG	F	G	ITERM
1	1421	1425	5688	0.465831486E−25	0.418E−12	3
2	39	45	200	0.231406390E−14	0.350E−06	4
3	17	18	108	0.839782900E−09	0.933E−06	4
4	24	25	100	269.499543	0.666E−10	4
5	11	12	72	0.795109456E−10	0.473E−06	4
6	13	16	196	0.125944855E−10	0.815E−06	4
7	12	13	78	336.937181	0.300E−06	4
8	4	5	90	761774.954	0.216E−06	4
9	7	9	16	316.436141	0.146E−06	4
10	69	75	630	−135.290000	0.291E−11	4
11	23	28	144	86.8673060	0.436E−07	4
12	27	29	112	982.273617	0.393E−06	4
13	6	7	28	0.598998674E−10	0.693E−06	4
14	2	3	18	0.129013604E−08	0.792E−06	4
15	9	10	40	1.92401599	0.414E−06	4
16	7	8	48	−427.404476	0.565E−07	4
17	8	9	54	−0.379921091E−01	0.314E−10	4
18	7	8	48	−0.245741193E−01	0.218E−09	4
19	6	7	42	59.5986241	0.952E−08	4
20	14	15	90	−1.00013520	0.139E−08	4
21	11	12	72	2.13866377	0.331E−08	4
22	30	34	186	1.00000000	0.164E−08	4
Σ	1767	1813	8060		TIME = 2.80	

Table 9. Results obtained by program TNEDU

Problem	NIT	NFV	NFG	F	G	ITERM
1	1420	1424	5680	0.00000000	0.000E+00	3
2	128	130	640	1980.05047	0.911E−10	4
3	17	19	108	0.189355864E−09	0.340E−06	4
4	10	12	44	269.522686	0.328E−09	4
5	13	15	84	0.391905635E−12	0.536E−06	4
6	13	14	196	0.136396633E−11	0.901E−06	4
7	30	32	186	336.920046	0.151E−05	2
8	37	38	684	761925.725	0.119E−06	4
9	507	508	1016	428.056916	0.347E−13	4
10	109	127	990	−80.4518214	0.639E−06	4
11	6	8	42	72291.4951	0.178E−08	4
12	519	520	2080	4994.21410	0.236E−06	4
13	3	4	16	0.660542076E−23	0.363E−11	3
14	2	3	18	0.129013604E−08	0.792E−06	4
15	9	10	40	1.92401599	0.414E−06	4
16	15	18	96	−427.391653	0.342E−06	4
17	8	9	54	−0.379921091E−01	0.314E−10	4
18	7	8	48	−0.245741193E−01	0.218E−09	4
19	13	16	84	1654.94525	0.174E−08	4
20	14	15	90	−1.00013520	0.139E−08	4
21	9	10	60	2.41354873	0.388E−08	4
22	30	34	186	1.00000000	0.164E−08	4
Σ	2919	2974	12442		TIME = 6.58	

Table 10. Results obtained by program TNEDS

Problem	NIT	NFV	NFG	F	G	ITERM
1	1447	1450	5792	0.173249493E−16	0.138E−06	3
2	79	89	400	0.169144088E−20	0.382E−09	3
3	18	19	114	0.180692317E−09	0.316E−06	4
4	24	25	100	269.499543	0.136E−08	4
5	11	12	72	0.990922474E−10	0.511E−06	4
6	17	21	252	0.166904871E−10	0.898E−06	4
7	11	12	72	336.937181	0.629E−06	4
8	6	11	126	761774.954	0.237E−05	2
9	7	8	16	316.436141	0.362E−08	4
10	70	74	639	−133.630000	0.221E−07	4
11	27	31	168	86.8673060	0.416E−06	4
12	28	29	116	982.273617	0.889E−11	4
13	7	8	32	0.402530175E−26	0.153E−13	3
14	2	3	18	0.129028794E−08	0.820E−06	4
15	10	11	44	1.92401599	0.217E−06	4
16	12	15	78	−427.404476	0.894E−09	4
17	8	9	54	−0.379921091E−01	0.391E−09	4
18	8	9	54	−0.245741193E−01	0.705E−10	4
19	7	8	48	59.5986241	0.106E−08	4
20	10	11	66	−1.00013520	0.277E−11	4
21	11	12	72	2.13866377	0.154E−06	4
22	46	51	282	1.00000000	0.376E−08	4
Σ	1866	1918	8615	NCG = 1033	TIME = 2.72	

Table 11. Results obtained by program TNECU

Problem	NIT	NFV	NFG	F	G	ITERM
1	1433	1438	5736	0.000000000	0.000E−00	3
2	218	241	1085	3918.57298	0.377E+01	4
3	17	19	108	0.191263604E−09	0.349E−06	4
4	10	12	44	269.522686	0.733E−08	4
5	13	15	84	0.309044401E−13	0.194E−06	4
6	13	14	196	0.328095928E−12	0.411E−06	4
7	19	27	120	337.413070	0.438E−06	4
8	37	38	684	761925.725	0.536E−06	2
9	643	644	1288	428.056916	0.432E−13	4
10	124	151	1125	−79.4726101	0.531E−07	4
11	6	8	42	72291.4951	0.188E−08	4
12	199	200	800	4994.21410	0.238E−06	4
13	4	5	20	0.650986116E−23	0.504E−11	3
14	2	3	18	0.129028794E−08	0.820E−06	4
15	10	11	44	1.92401599	0.217E−06	4
16	14	17	90	−427.391653	0.114E−12	4
17	8	9	54	−0.379921091E−01	0.391E−09	4
18	8	9	54	−0.245741193E−01	0.705E−10	4
19	13	16	84	1654.94525	0.155E−08	4
20	10	11	66	−1.00013520	0.277E−11	4
21	9	10	60	2.41354873	0.517E−06	4
22	46	51	282	1.00000000	0.376E−08	4
Σ	2900	2989	12304	NCG = 1877	TIME = 6.28	

Table 12. Results obtained by program TNECS

Problem	NIT	NFV	NFG	F	G	ITERM
1	2654	3627	3627	0.794789730E−16	0.213E−06	3
2	105	179	179	83.3161404	0.498E−06	4
3	40	45	45	0.267007684E−12	0.823E−06	4
4	37	45	45	269.499543	0.605E−06	4
5	16	17	17	0.106026711E−11	0.728E−06	4
6	38	40	40	0.546961387E−11	0.882E−06	4
7	22	26	26	335.252624	0.105E−06	4
8	26	40	40	761774.954	0.295E−04	2
9	193	202	202	316.436141	0.155E−05	2
10	227	258	258	−125.810000	0.351E−04	2
11	100	127	127	10.7765879	0.566E−06	4
12	28	29	29	982.273617	0.102E−06	4
13	1	2	2	0.00000000	0.000E+00	3
14	25	28	28	0.104289352E−08	0.927E−06	4
15	8	15	15	1.92401599	0.482E−07	4
16	25	35	35	−427.404476	0.130E−06	4
17	15	17	17	−0.379921091E−01	0.141E−06	4
18	5	11	11	−0.245741193E−01	0.311E−07	4
19	19	23	23	59.5986241	0.466E−06	4
20	37	97	97	−1.00013520	0.212E−08	4
21	37	40	40	2.13866377	0.767E−06	4
22	55	211	211	1.00000000	0.610E−07	4
Σ	3713	5114	5114		TIME = 4.28	

Table 13. Results obtained by program TSEDU

Problem	NIT	NFV	NFG	F	G	ITERM
1	2591	3322	3322	0.00000000	0.000E+00	3
2	344	347	347	35.1211309	0.107E−06	4
3	39	43	43	0.441691821E−12	0.425E−06	4
4	21	22	22	269.522686	0.105E−06	4
5	16	17	17	0.783032535E−11	0.279E−06	4
6	32	33	33	0.959526458E−11	0.801E−06	4
7	19	21	21	337.722479	0.247E−06	4
8	52	56	56	761925.725	0.780E−04	2
9	1001	1003	1003	428.056916	0.192E−06	4
10	191	222	222	−86.7038382	0.225E−05	2
11	13	18	18	72291.4951	0.285E−08	4
12	228	235	235	4994.21410	0.304E−06	4
13	1	2	2	0.00000000	0.000E+00	3
14	25	28	28	0.104289352E−08	0.927E−06	4
15	8	15	15	1.92401599	0.534E−07	4
16	21	22	22	−427.391653	0.759E−06	4
17	15	17	17	−0.379921091E−01	0.299E−06	4
18	5	10	10	−0.245741193E−01	0.193E−07	4
19	20	25	25	1654.94525	0.351E−06	4
20	78	130	130	−1.00013520	0.196E−06	4
21	27	31	31	2.41354873	0.202E−06	4
22	52	190	190	1.00000000	0.418E−06	4
Σ	4799	5809	5809		TIME = 8.14	

Table 14. Results obtained by program TSEDS

Problem	NIT	NFV	NFG	F	G	ITERM
1	2606	3566	3566	0.650526028E−17	0.969E−07	3
2	108	177	177	111.363179	0.853E−05	2
3	40	45	45	0.267007684E−12	0.823E−06	4
4	37	45	45	269.499543	0.605E−06	4
5	16	17	17	0.106026711E−11	0.728E−06	4
6	38	40	40	0.546961387E−11	0.882E−06	4
7	26	31	31	335.252624	0.315E−06	4
8	25	39	39	761774.954	0.149E−03	2
9	191	210	210	316.436141	0.115E−05	2
10	233	264	264	−121.691827	0.190E−04	2
11	113	144	144	10.7765879	0.649E−07	4
12	28	29	29	982.273617	0.103E−06	4
13	1	2	2	0.00000000	0.000E+00	3
14	25	28	28	0.104289348E−08	0.927E−06	4
15	27	41	41	1.92401599	0.668E−07	4
16	25	35	35	−427.404476	0.263E−06	4
17	15	17	17	−0.379921091E−01	0.141E−06	4
18	8	12	12	−0.245741193E−01	0.358E−11	4
19	19	23	23	59.5986241	0.466E−06	4
20	42	74	74	−1.00013520	0.965E−09	4
21	37	40	40	2.13866377	0.767E−06	4
22	48	192	192	1.00000000	0.107E−06	4
Σ	3708	5071	5071	NCG = 48642	TIME = 7.64	

Table 15. Results obtained by program TSECU

Problem	NIT	NFV	NFG	F	G	ITERM
1	2598	3347	3347	0.00000000	0.000E+00	3
2	352	361	361	35.1211309	0.853E−05	2
3	39	43	43	0.441691822E−12	0.425E−06	4
4	21	22	22	269.522686	0.105E−06	4
5	16	17	17	0.783032535E−11	0.279E−06	4
6	32	33	33	0.959526458E−11	0.801E−06	4
7	19	21	21	337.722479	0.162E−05	2
8	46	49	49	761925.725	0.792E−04	2
9	1001	1003	1003	428.056916	0.348E−08	4
10	203	233	233	−86.7188428	0.288E−04	2
11	21	38	38	72291.4951	0.135E−10	4
12	223	230	230	4994.21410	0.303E−06	4
13	1	2	2	0.00000000	0.000E+00	3
14	25	28	28	0.104289348E−08	0.927E−06	4
15	17	27	27	1.92401599	0.553E−07	4
16	21	22	22	−427.391653	0.759E−06	4
17	15	17	17	−0.379921091E−01	0.299E−06	4
18	8	12	12	−0.245741193E−01	0.358E−11	4
19	20	25	25	1654.94525	0.351E−06	4
20	33	46	46	−1.00013520	0.959E−10	4
21	27	31	31	2.41354873	0.202E−06	4
22	51	185	185	1.00000000	0.834E−06	4
Σ	4789	5792	5792	NCG = 15187	TIME = 6.66	

Table 16. Results obtained by program TSECS

Problem	NIT	NFV	NFG	F	G	ITERM
1	3124	3134	3134	0.287703261E−08	0.582E−08	4
2	286	287	287	0.379499216E−08	0.203E−06	2
3	71	71	71	0.233196848E−09	0.100E−07	4
4	40	40	40	126.863549	0.699E−08	4
5	282	282	282	0.732927514E−07	0.400E−08	4
6	344	344	344	0.836329152E−08	0.326E−08	4
7	286	287	287	2391.16999	0.673E−04	2
8	457	458	458	0.478407013E−05	0.682E−08	4
9	2514	2516	2516	552.380551	0.448E−08	4
10	907	907	907	131.888476	0.579E−08	4
11	269	271	271	0.173668302E−09	0.266E−08	4
12	1805	1810	1810	621.128947	0.906E−02	2
13	680	681	681	2940.50943	0.140E−03	2
14	538	539	539	112.314954	0.541E−08	4
15	364	364	364	36.0935676	0.986E−08	4
16	1004	1004	1004	13.2000000	0.904E−08	4
17	380	380	380	0.268534232E−01	0.871E−09	4
18	15319	15321	15321	0.589970806E−08	0.925E−08	4
19	3972	4056	4056	0.565862690E−08	0.887E−08	4
20	774	988	988	0.406495193E−08	0.468E−08	4
21	247	248	248	264.000000	0.364E−03	2
22	1191	1192	1192	593.360762	0.145E−03	2
Σ	34854	35180	35180		TIME = 14.86	

Table 17. Results obtained by program TSENU

Problem	NIT	NFV	NFG	F	G	ITERM
1	1377	1379	1379	0.697391982E−22	0.130E−09	3
2	41	46	46	0.216572157E−16	0.154E−06	3
3	11	12	14	0.136731713E−09	0.233E−06	4
4	13	16	21	134.749772	0.279E−06	4
5	4	5	7	0.111058357E−10	0.887E−06	4
6	6	7	13	0.742148235E−26	0.303E−12	3
7	10	12	23	60734.8551	0.648E−07	4
8	21	26	24	0.253357740E−08	0.800E−06	4
9	15	16	36	2216.45871	0.104E−10	4
10	12	18	21	191.511336	0.524E−07	4
11	2587	2593	2649	0.647358980E−27	0.359E−12	3
12	16	20	23	19264.6341	0.513E−10	4
13	17	21	28	131234.018	0.784E−08	4
14	5	8	18	108.517888	0.227E−08	4
15	6	7	15	18.1763146	0.290E−06	4
16	15	21	40	2.51109677	0.724E−06	4
17	15	20	19	0.257973699E−16	0.275E−08	3
18	42	44	45	0.151517993E−24	0.122E−10	3
19	15	16	23	0.354943701E−14	0.255E−06	4
20	40	47	43	0.978394865E−18	0.113E−10	3
21	10	11	17	647.828517	0.773E−11	4
22	26	32	45	4486.97024	0.602E−07	4
Σ	4304	4377	4549		TIME = 4.59	

Table 18. Results obtained by program TGADU

Problem	NIT	NFV	NFG	F	G	ITERM
1	1011	1013	1013	0.00000000	0.000E+00	3
2	260	273	508	1959.28649	0.439E−12	4
3	10	12	13	0.784354965E−09	0.868E−06	4
4	14	18	19	134.761343	0.827E−08	4
5	4	5	7	0.438081882E−11	0.697E−06	4
6	6	7	13	0.791460684E−17	0.934E−08	3
7	22	23	61	145814.000	0.000E+00	4
8	25	32	28	0.978141069E−06	0.782E−06	4
9	44	45	153	2220.17880	0.181E−09	4
10	12	19	21	191.511336	0.301E−07	4
11	3977	2992	2990	0.00000000	0.000E+00	3
12	29	30	50	67887.2385	0.438E−12	4
13	19	20	36	147906.000	0.000E+00	4
14	1	2	6	126.690556	0.000E+00	4
15	24	27	81	18.1763146	0.203E−10	4
16	46	50	135	3.59074140	0.470E−10	4
17	11	12	15	0.969524252E−21	0.171E−10	3
18	0	1	3	0.00000000	0.000E+00	3
19	26	30	34	0.202602070E−14	0.193E−06	4
20	929	930	2780	498.800124	0.359E−05	2
21	20	21	33	649.598077	0.280E−08	4
22	24	31	55	4488.96148	0.242E−07	4
Σ	6514	5593	8054		TIME = 8.08	

Table 19. Results obtained by program TGADS

Problem	NIT	NFV	NFG	F	G	ITERM
1	1108	1110	1110	0.00000000	0.000E+00	3
2	602	617	636	66.1697783	0.237E−11	4
3	11	12	14	0.199275556E−09	0.208E−06	4
4	11	13	17	134.749772	0.592E−07	4
5	4	5	7	0.116836300E−10	0.908E−06	4
6	6	7	13	0.786363292E−26	0.311E−12	3
7	17	40	29	60734.8551	0.428E−05	6
8	22	25	25	0.127726626E−07	0.160E−06	4
9	13	14	38	2216.45871	0.240E−11 4	
10	129	147	176	191.511336	0.104E−07	4
11	3010	3016	3012	0.402368464E−24	0.902E−11	3
12	205	226	236	22287.9069	0.452E−08	4
13	123	132	152	131234.018	0.744E−09	4
14	7	8	32	108.517888	0.148E−07	4
15	13	20	42	18.1763146	0.445E−05	2
16	13	14	34	2.51109677	0.389E−10	4
17	23	27	27	0.660798236E−09	0.741E−06	4
18	49	53	52	0.119511868E−21	0.344E−09	3
19	15	16	23	0.339085307E−13	0.788E−06	4
20	17	18	32	0.110442500E−09	0.881E−07	4
21	15	18	23	647.696136	0.262E−06	4
22	47	59	98	4486.97024	0.663E−08	4
Σ	5460	5597	5828	NCG = 6229	TIME = 4.70	

Table 20. Results obtained by program TGACU

Problem	NIT	NFV	NFG	F	G	ITERM
1	1028	1031	1030	0.00000000	0.000E+00	3
2	263	262	511	1959.28649	0.881E−10	4
3	10	12	13	0.815962946E−09	0.899E−06	4
4	11	14	17	134.761343	0.206E−10	4
5	4	5	7	0.438081882E−11	0.697E−06	4
6	6	7	13	0.791460667E−17	0.934E−08	3
7	15	16	42	145814.000	0.000E+00	4
8	17	18	20	0.201167216E−07	0.162E−06	4
9	54	55	203	2220.17880	0.614E−10	4
10	95	105	126	191.511336	0.527E−06	4
11	4577	3591	3587	0.00000000	0.000E+00	3
12	49	50	84	67887.2385	0.351E−07	4
13	18	19	41	147906.000	0.000E+00	4
14	1	2	6	126.690556	0.000E+00	4
15	33	72	85	18.1763146	0.182E−04	6
16	8	12	29	3.59074140	0.542E−06	4
17	22	24	26	0.526524970E−09	0.781E−06	4
18	0	1	3	0.00000000	0.000E+00	3
19	28	32	36	0.209831734E−13	0.620E−06	4
20	932	933	2791	498.800124	0.576E−12	4
21	21	22	34	649.598077	0.324E−08	4
22	44	51	89	4488.96148	0.645E−10	4
Σ	7236	6334	8793	NCG = 11287	TIME = 6.44	

Table 21. Results obtained by program TGACS

Problem	NIT	NFV	NFG	C	G	ITERM
1	53	66	54	0.260681654E−14	0.120E−07	4
2	107	160	108	0.502441303E−12	0.262E−06	4
3	33	41	34	0.535439934E−08	0.814E−06	4
4	51	93	52	0.540217976	0.383E−06	4
5	23	24	24	0.132910215E−08	0.482E−06	4
6	46	52	47	0.216701250E−08	0.436E−06	4
7	48	113	49	0.260162540	0.430E−06	4
8	21	58	22	282.380956	0.806E−06	4
9	59	146	60	0.185849706	0.287E−06	4
10	159	215	160	−0.251638288	0.250E−06	4
11	70	96	71	0.538829394E−01	0.139E−07	4
12	136	245	137	0.941962422	0.207E−06	4
13	2	4	3	0.456380111E−19	0.304E−11	3
14	5	6	6	0.162409086E−08	0.671E−06	4
15	116	120	117	0.199003538E−01	0.436E−06	4
16	110	214	111	−0.388943896E−02	0.518E−05	2
17	33	49	34	−0.110417781E−06	0.764E−06	4
18	62	77	63	0.744234285E−09	0.611E−07	4
19	9	23	10	42.6746811	0.373E−09	4
20	25	32	26	−0.497512435E−02	0.422E−06	4
21	16	23	17	0.298487756E−01	0.595E−06	4
22	32	82	33	0.577532726E−02	0.972E−07	4
Σ	1216	1939	1238		TIME = 0.75	

Table 22. Results obtained by program TMAXU

Problem	NIT	NFV	NFG	C	G	ITERM
1	337	355	338	0.193178806E−13	0.254E−05	3
2	127	151	128	0.120336380E−12	0.424E−05	3
3	25	28	26	0.383710546E−09	0.331E−06	4
4	67	77	68	126.863549	0.358E−02	2
5	6	7	7	0.494049246E−14	0.666E−07	3
6	13	17	14	0.663150090E−13	0.141E−06	3
7	73	108	74	2391.16999	0.209E+00	2
8	241	243	242	0.383133726E−07	0.708E−06	4
9	816	982	5712	550.200389	0.856E−04	6
10	84	106	85	131.888475	0.242E−05	2
11	732	751	733	0.799693645E−12	0.581E−05	3
12	203	237	204	612.723020	0.601E−04	2
13	90	111	91	2940.50941	0.245E−03	2
14	84	107	85	112.314955	0.480E−05	2
15	56	102	56	36.0935678	0.927E−04	6
16	67	108	67	13.2000005	0.139E−03	6
17	337	344	338	0.100472795E−13	0.167E−06	3
18	2024	2239	2025	0.00000000	0.000E+00	3
19	23	24	24	0.938360500E−12	0.217E−04	3
20	22	44	22	0.121058719E−11	0.398E−05	6
21	65	90	66	262.921649	0.108E−05	2
22	608	627	609	593.367735	0.525E−02	2
Σ	6100	6858	11014		TIME = 3.08	

Table 23. Results obtained by program TSUMU

Problem	NIT	NFV	NFG	F	G	ITERM
1	10	41	0	0.224531E−22	0.168207E−07	3
2	9	46	0	0.106897E−22	0.163517E−06	3
3	3	19	0	0.333989E−19	0.223053E−06	3
4	7	23	0	0.348196E−17	0.177085E−02	3
5	12	63	0	0.117206E−16	0.694210E−06	3
6	17	52	0	0.110919E−16	0.167579E−11	3
7	13	41	0	0.339913E−19	0.457009E−03	3
8	13	73	0	0.125748E−25	0.193922E−04	3
9	13	99	0	0.432936E−21	0.201706E−03	3
10	5	41	0	0.803846E−25	0.415983E−03	3
11	12	37	0	0.189327E−25	0.423583E−05	3
12	18	55	0	0.129272E−16	0.713317E−13	3
13	18	39	0	0.105290E−16	0.341327E−13	3
14	4	13	0	0.774783E−20	0.441968E−05	3
15	5	36	0	0.182567E−17	0.471251E−03	3
16	53	319	0	0.462169E−17	0.153957	3
17	14	48	0	0.449140E−22	0.105525E−03	3
18	26	79	0	0.977445E−16	0.245792E−01	3
19	2	7	0	0.309324E−21	0.370062E−09	3
20	13	43	0	0.428279E−20	0.203421E−07	3
21	12	37	0	0.200623E−20	0.255404E−10	3
22	7	50	0	0.195350E−19	0.106707E−05	3
23	29	262	0	0.390327E−17	0.200697E−10	3
24	6	31	0	0.822526E−23	0.812457E−09	3
25	9	46	0	0.147127E−23	0.395357E−09	3
26	12	61	0	0.608837E−17	0.420862E−07	3
27	10	51	0	0.275078E−20	0.121824E−06	3
28	10	60	0	0.229532E−16	0.213811E−05	3
29	4	53	0	0.124549E−19	0.130673E−05	3
30	12	162	0	0.222959E−21	0.107876E−07	3
Σ	378	1987	0	NCG = 1543	TIME = 3.94	

Table 24. Results obtained by program TEQNU

Problem	NIT	NFV	NFG	F	G	ITERM
1	30	64	0	0.326079E−18	0.154142E−03	3
2	17	57	0	0.720058E−19	0.261551E−07	3
3	5	11	0	0.861220E−16	0.366389E−03	3
4	11	19	0	0.115060E−18	0.358897E−01	3
5	24	59	0	0.718350E−16	0.564797E−06	3
6	22	31	0	0.167377E−16	0.898624E−08	3
7	25	42	0	0.137004E−20	0.185851E−05	3
8	21	60	0	0.496243E−28	0.183782E−07	3
9	32	71	0	0.220876E−21	0.800603E−05	3
10	9	24	0	0.202316E−20	0.162996E−03	3
11	16	23	0	0.116022E−21	0.130018E−02	3
12	23	40	0	0.861690E−16	0.190460E−08	3
13	24	32	0	0.234892E−16	0.204525E−08	3
14	8	13	0	0.596974E−21	0.811563E−05	3
15	12	28	0	0.124901E−17	0.305897	3
16	22	78	0	0.984840E−20	0.125407E−03	3
17	17	43	0	0.130235E−20	0.154659E−04	3
18	46	61	0	0.224793E−17	0.116353E−01	3
19	2	5	0	0.704403E−18	0.221630E−06	3
20	18	30	0	0.158787E−16	0.312477E−03	3
21	25	34	0	0.233925E−16	0.135133E−05	3
22	14	45	0	0.189862E−17	0.128826E−01	3
23	23	106	0	0.194742E−18	0.550497E−08	3
24	20	53	0	0.737500E−17	0.611156E−08	3
25	29	50	0	0.208794E−17	0.413643E−08	3
26	36	67	0	0.132055E−17	0.481013E−08	3
27	40	75	0	0.659356E−17	0.862034E−08	3
28	27	83	0	0.461856E−18	0.268680E−08	3
29	12	95	0	0.206962E−16	0.754042E−08	3
30	18	145	0	0.740533E−16	0.167985E−07	3
Σ	628	1544	0	NCG = 1331	TIME = 3.22	

Table 25. Results obtained by program TEQLU

REFERENCES

[1] Lukšan L., Vlček J. Sparse and partially separable test problems for unconstrained and equality constrained optimization. Report V-767, Prague, ICS AS CR, 1998.