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Reference levels in School Mathematics Education in Europe

National Presentation

SWEDEN

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1. General description of the mathematics teaching context

1.1 Description of the school system

The school system in Sweden is regulated by the Education Act. It states that education should be of equivalent value, irrespective of where in the country and who provides the education. The central steering takes place by means of national goals, guidelines and by national assessment. The main central authority is the National Agency for Education, responsible for monitoring and evaluating all school activities and of inspecting schools. The Agency also has the responsibility of providing national tests and guidelines for assessing these.

The municipalities operate most schools. However there is a growing but still low number of private schools. About 3% of all pupils go to private and independent schools, about one third of these students to schools based on pedagogues such as Maria Montessori or Rudolf Steiner and one third to confessional schools (National Agency of Education, 2000).

The municipality is obliged to create a school plan and to monitor and evaluate local schools and to provide teachers with in-service training. The school plan states the specific goals of the schools of the municipality.

In each school, teachers are obliged to decide on a working plan for each subject, stating the means by which the national goals are to be attained. The local freedom and responsibility is great, when it comes to the organising of teaching and learning, and to allocate teaching time between different grades.

1.1.1 Compulsory school

Compulsory education has the form of a 9-year comprehensive school (grundskola) for children aged 7 - 16. A child enters the first school year (year 1) in August the calendar year when he/she attains seven years. Therefore children are approximately between 6.5 years and 7.5 years at school start. The pupils are approximately between 15.5 years and 16.5 years when receiving their leaving certificate from compulsory school after finishing year 9.

Since 1991 the municipality may offer parents to let their children start school one year earlier, that is at the age of 6 years. All municipalities now offer this option to parents. A child entering school in year ”0” (pre-school class) normally would spend ten years in compulsory school. Most children, 93%, now attend this introductory year (National Agency of Education, 2000).

Compulsory school is not formally divided into levels, but most schools would receive children in year 1 - 6 (primary schools, “låg- och mellan-stadium”) or children in year 7 - 9 (lower secondary schools, “högstadium”).

In 1994 the Parliament adopted new curricular guidelines for the whole school system, geared to a new objective and result-related governing system for schools.

(Swedish Ministry of Education and Science, 1994). The curriculum sets out basic values of the school and provides objectives and guidelines. The national syllabus for each subject states the goals, which are to be attained by all pupils at the end of the fifth and the ninth year. It also states the goals to strive towards in each subject. Criteria for the grades Pass and Pass with credit are defined in subject specific but rather general terms. (National Agency for Education, 1996,b). The teachers have to define criteria for the highest degree, Pass with distinction at the local level.

For municipal schools there is a timetable, defined in the Education Act. It indicates a minimum of teacher-led instruction for each of 11 subjects or group of subjects, in total 6 665 hours of 60 minutes during the nine years. Swedish is the largest subject with 1490 hours. Mathematics is the second largest with 900 hours.

Each pupil has an individual choice of 470 hours and each school defines a choice of the school, 410 hours. These hours, or part of them, may be devoted to mathematics, although this is not common.

The assessment is the responsibility of the local school and the individual teacher. To their help and to ensure comparable grading on a national level there are national examinations in the subjects Swedish, English and mathematics. National tests are administered during the fifth (voluntarily for the school) and the ninth (compulsory) year in these subjects. Guidelines are provided for the grades Pass and Pass with credit. The national tests transform the general criteria of the syllabi to something more concrete.

Final awards in each subject will be on a three-point scale: Pass, Pass with credit and Pass with distinction. For those students who do not attain Pass, no award is given. Teachers must assess every pupil regularly and identify his/her weaknesses and strengths. Each semester one teacher (the home-group teacher) gives detailed feedback during an informative talk to each pupil and his/her parents about the student's progress in all subjects. No marks are given before year eight, but marks are given in all subjects every semester beginning year eight.

It is the responsibility of the municipality and the school to give every pupil individual support if needed in order to attain Pass grade in each subject at the end of the ninth year.

1.1.2 Post-compulsory education - upper secondary level

Upper secondary schooling is not compulsory, but 98% of the pupils leaving year 9 of the comprehensive school continue to the upper secondary level (National Agency of Education, 2000). Municipalities are obliged to offer upper secondary schooling for all pupils leaving compulsory school.

School at this level is organised in one legal and administrative unit ("gymnasieskola") that offers a number of different programmes. This unified structure was introduced in the 1992-93 school year in small scale and implemented all over the country the year 1993-94. All programmes whether they are vocational or preparing for further studies, are of three years duration. A student in any of the national programme, also the vocational ones, will meet the basic eligibility requirements for tertiary studies at universities or university colleges after having finished the programme. A requirement is Pass grade in courses that constitutes at least 90% of the upper secondary education (Swedish Ministry of Education and Science (1994).

There are 16 national programmes, two preparing for further studies and 14 vocational. The Natural science programme is directed towards further studies in mathematics, science subjects and technology. It gives the student the specific eligibility requirements for tertiary studies in these and most other areas. The Social science programme is directed towards further studies in social sciences, humanities, languages and economics. It gives the student the specific eligibility requirements for tertiary studies in these areas, according to choice of branch in the second and third year.

All national programmes have a number of specialised branches in year two and three.

Besides the national programmes, schools can offer specifically designed programmes. In order to be eligible for upper secondary school pupils are required to have Pass grades in Swedish, English and mathematics from compulsory school.

Students who do not fulfil the requirement are offered individual programmes of varying length and content.

Municipalities must offer a comprehensive selection of national programmes. Admission capacity for various programmes should be adjusted to pupils’ preferences. The great majority of pupils are admitted at their first hand choice of programme. The proportion of pupils admitted to different programmes the year 1998 were as follows: Social science 23%, Natural science 19%, vocational national programmes (14 programmes) 37%, special programmes 7%, individual programmes 13% (National Agency of Education, 1999 b).

Within the upper secondary school there are a number of courses that build upon each other in each subject. The volume of a course is indicated by a number of study points ranging between 30 and 120. The total amount of points for three years of study is between 2150 and 2300. The number of points corresponds approximately to the same number of teaching hours (one teaching hour equals 60 minutes). Each course is graded in three levels, and grades from all courses make up the leaving certificate.

There are a number of core courses, common to all programmes and in total corresponding to 680 points. Among these is one mathematics course, Mathematics A of 110 points.

Each programme also has a number of foundation subjects, specific for that programme. In each branch of a programme there are a number of compulsory courses offered in the second and third years. On top of that each student has a free choice of a number of courses.

National tests are compiled in certain subjects, among them mathematics. These are optional for schools to administer. However teachers are recommended to use the tests as a means to attain a grading conforming to the national level.

1.2 Place and importance of mathematics in the curriculum

1.2.1 Mathematics – the second largest subject

As stated above, mathematics is the second largest subject in compulsory school. Out of 6665 hours in total, 900 hours or 14% is devoted to mathematics. Teachers, pupils as well as parents regard mathematics as a most important subject.

However many pupils are struggling with the subject. Mathematics has the highest level of pupils not awarded pass degree at the end of the ninth year. 14% of all pupils did not attain Pass grade in the national test for year 9 in 1999 while the corresponding proportions in Swedish and English were 4% (National Agency for Education, 1999,d). Out of all students 6% left compulsory school without a Pass grade in mathematics 1999 (National Agency of Education, 2000).

In upper secondary school, the importance of the subject is still great. Swedish is the largest subject while English and mathematics are the second largest subjects among the core courses.

Sweden applies a system with very late differentiation in school mathematics. Due to the system with a common course Mathematics A at upper secondary level differentiation takes place at age 16-17 years. The total amount of teaching hours of mathematics common for all pupils throughout the school system consists of 900 hours at compulsory school and 110 points (approximately hours) of Mathematics A at the upper secondary level. That corresponds to approximately 11% of all teaching time year 1 - 12.

However, there is less stress on specialist mathematics. Most mathematics is offered at the Natural science programme. Four courses follow Mathematics A namely Mathematics B - E, in total 190 points. Only Mathematics B - D (130 points) are compulsory for the Natural science programme. A student leaving the Natural science programme with level E (most common) has devoted approximately 14% of their scheduled time to mathematics throughout school.

The specialist mathematics forms as little as 16% of the total amount of mathematics for a student leaving upper secondary school with the E level of mathematics. 84% corresponds to mathematics common to all students. The corresponding proportions for students at level D are 11% specialised mathematics and 89% mathematics in common with pupils in all other programmes.

Mathematics B and C (90 points together) are compulsory for the Social science programme, with an exception for the branch for Humanities, where only Mathematics B (40 points) is mandatory.

The great majority of schools offer extra courses to pupils who wish to take optional mathematics.

Most students aiming at tertiary studies reach the level C, D or E in mathematics.

1.2.2 Focus on weak students

It is of utmost importance for the school and the municipality to reduce the proportion of pupils who do not get Pass degree in the three subjects Swedish, English and mathematics when leaving compulsory school. These degrees define the requirements for entering one of the national programmes at upper secondary level. There is no job market for young persons of 16 years so it is of great political importance that all young can attend school until the age of 18. It is also of great importance for the individual to get an upper secondary education in order to be admitted to the job market later on. The number of unqualified jobs is diminishing very fast.

Hence, for political and other reasons, the focus for schools and municipalities seeking to improve their results is on those pupils who risk not to reach Pass degree. The fact that many other pupils do not attain a reasonable level of knowledge in mathematics, considering their aptitude, seems to be of less importance.

1.2.3 Differentiation

(*Remark*: I use the word *stream* to indicate a system where pupils are divided into streams according to academic performance. Different goals and contents may be applied to each stream. Assessment is independent for each stream.

I use the word *tracking* to indicate a teaching method, where all pupils follow the same course but teaching is delivered to pupils divided into ability groups. Goals and assessment are common for all tracks. Tracking normally would be combined with possibilities to move from time to time to a lower or higher track, while streaming would not allow a shift from a lower to a higher stream without some completion work.)

1.2.3.1 Compulsory school

The question whether streaming should be applied in mathematics during year 7 - 9, has been debated since the comprehensive school was introduced in 1962 and before that during a period of experiments with organisational models in the 50-ties (National Agency for Education, 1997). The question has turned into a political rather than a pedagogical one, and has been discussed with some harshness from time to time. Until 1994, two different streams (levels) existed in mathematics (as well as in English). Teaching groups in mathematics were split according to these two levels, starting in year 7. Assessment and grading was done independently for the two streams/levels.

From 1994 there are no longer different streams. All pupils study the same course. Goals, content and assessment are identical. It is the responsibility of the head of each school to decide whether different tracks should be used. There is not yet any investigation made in order to identify the proportion of schools using tracks in mathematics. However it seems clear that tracking is quite common. It is also common to use no tracking, but rather try to individualise within the teaching group according to each student's ability. Some of the widely used textbooks publish two versions of material aimed at year 8 and 9, one for motivated students and one for slower learners. All textbooks have some kind of differentiation according to results at diagnostic tests in each chapter.

The most common tracking system seems to consist of two or three tracks, even though there are examples of more levels. One example is the following. The weakest group get intense teacher support in small groups. The ”middle” and largest group of pupils work in groups of 20 to 25, while the most advanced pupils may have to manage in a larger group, up to 35. In this way the school seeks to give each pupil the amount of teacher aid that they need. Pupils can move from one track to another at certain times, if it is judged to be in the best interest of the pupil.

1.2.3.2 Post-compulsory school

The debate about differentiation became intense again during the preparation for a new upper secondary school during the beginning of the nineties. Again it was a political decision to have a basic course, Mathematics A, common for all programmes, for vocational as well as for those preparing for further studies. The decision that the core subjects, among them Mathematics A, would be common for all upper secondary students was taken by the Parliament.

Upper secondary schools are now free to allocate teaching time according to the specific needs of a group of pupils. Most schools allocate less teaching time for Mathematics A in the Natural science programme, more in the Social science programme and even more in the vocational programmes. The number of teaching hours may vary between 60 and 110+, while the number of study points is always the same, 110 points.

Despite this effort to meet the needs of all pupils the proportion of pupils that fail mathematics is high in some of the programmes. In the Natural science programme only 1% failed the test on Mathematics A, timed part. In some of the vocational programmes between 20% and 60% failed the test (timed part) (Kjellström Katarina,1996). A less, but still large proportion of students in the vocational programmes does not pass Mathematics A. In a couple of programs this proportion is as high as 17% in 1999. (National Agency of Education, 2000). This fact of course constitutes a great problem.

2. Main mathematics objectives

2.1 Compulsory school

Objectives and goals are laid down in the curriculum and the syllabus for mathematics (National Agency of Education, 1996, b). The syllabus for mathematics opens with a general description of characteristic traits of mathematics as a field of knowledge.

The main overall objectives for learning in compulsory school mathematics are the following:

Students should

- gain confidence in their own mathematical thinking
- understand and be able to use basic concepts and methods
- be able to solve problems with the use of mathematics
- be able to use calculators and computers with good judgement
- communicate mathematics both in writing and verbally
- be able to create and critically evaluate simple mathematical models.

Some knowledge and understanding of the history of mathematics and the role of mathematics in society are also mentioned among the general goals.

These overall objectives require pupils to develop good understanding in the fields of arithmetic, geometry, statistics and algebra. They also demand from pupils some basic knowledge of the concepts of function and probability. The knowledge is specified in more detail as a list of competencies in each field defining goals to strive towards for every pupil. These goals are to be interpreted as aiming high and leaving room for great individual variation.

Besides the “goals to strive towards” there is also a minimum level of knowledge defined in the form of “goals to attain”.

The goals to attain at the end of year nine are the following:

The pupil should have

- the mathematical knowledge and skills required for everyday life, for life as a citizen and for further studies

- an understanding of whole and rational numbers
- a good skill in estimation and computing with natural numbers, numbers in decimal form, percentage and proportionality
- a good command of methods for measuring and comparing lengths, area, volumes, angles, masses, time intervals.

The pupils should also be able to

- recognise and describe properties of common geometrical objects and be able to interpret scales and maps
- interpret, put together and analyse statistical data in tables and diagrams
- use the concept of probability in simple cases of randomness
- formulate and use simple formulas and equations for solving problems
- interpret and use graphs of some functions.

These goals to attain at the end of year nine define the lowest acceptable level of knowledge and understanding for a pupil to get Pass degree while leaving the compulsory school. According to the goals to strive towards, most students should reach a higher level.

The goals defined in the curriculum and syllabus are supplemented by local goals, decided at each school.

2.2 Upper secondary level

The syllabus (National Agency of Education, 1995 a and b) opens with a statement that knowledge of mathematics is a competence required for studies in other subjects and for further studies at tertiary level. The general objectives of mathematics at upper secondary level are similar to the goals of the compulsory school but they are formulated in a slightly different way. There is more stress on the ”mathematics world”. The first objective again is that the student should gain confidence in his/her own thinking as well as in his/her ability to learn and use mathematics. Then follows

> ”The course is designed towards students experiencing satisfaction in mastering mathematical concepts and methods, in discovering patterns and relationships, and solving problems, as well as learning to use and realise the value of mathematical symbols and expressions. It is essential that students learn to understand and conduct mathematical reasoning, create and use mathematical models and critically examine their conditions, possibilities and limitations, as well as learning how to give account of their thoughts, both verbally and in writing.” (National Agency for Education, 1995 a and b).

The aim of Mathematics A is to deepen the pupils understanding of concepts and improve the mastering of methods appearing already in compulsory school.

The aims of courses B - E are to give understanding and mastering of new methods in various areas, described in the next section.

The learning of an efficient use of calculators is a general goal.

As in the case of the compulsory school, national tests make the goals more concrete. Tests are given for the courses A, C, D and E, but they are not compulsory for schools to use.

2.3 National assessment in mathematics

The curriculum builds upon a view that teachers are trained and qualified enough to translate the goals into a concrete study plan for their pupils. But many teachers find the goals stated in the curriculum and the syllabus for mathematics vague.

The goals of the curriculum and syllabus are interpreted and transformed into something more concrete in the national tests. The tests are compiled and distributed by the PRIM-group at Stockholm Institute of Education (primary and lower secondary level and Mathematics A at upper secondary level) and the Unit for Pedagogical

Measurement at Umeå University (other courses at upper secondary level) on commission of the National Agency for Education.

In compulsory school national tests are administered during the fifth (voluntarily for schools to use) and ninth (compulsory) years. Guidelines are provided for the grades Pass and Pass with credit. Diagnostic material in mathematics for grades two and seven are also provided by the PRIM-group.

National tests are compiled every year for the courses Mathematics A, C, D and E at upper secondary level. These are optional for schools to administer. However teachers are recommended to use the tests as a means to conform their grading to a national norm.

All national tests consist of one or two individual and timed parts with items requiring short answers or solutions. Some of the questions might not have just one correct answer, but rather ask for one of several possible solutions. There are also non-timed parts with open ended and more complex problem. This gives the teacher a possibility to assess pupils' competence to structure their work while solving a more complex type of assignment. Some of the tests also include an oral part. In compulsory school, there is also one part in each test that includes working in pair or groups.

The non-traditional features of the national tests are there partly in order to influence teachers' methods and to spread new teaching ideas.

3. Basic content

3.1 Compulsory school

Four main areas are mentioned in the syllabus: arithmetic, geometry, statistics and algebra. Besides those probability and functions are also mentioned. The content is not prescribed in any detail in the central steering documents. For example there is no mentioning of the square, rectangle, triangle or circle in the mathematics syllabus. Instead the wordings are:

"... properties of common geometrical objects" (goals to attain)

".... understand and use basic geometrical concepts, properties, relations and theorems" (goals to strive towards).

The content is partly defined by the national tests, partly by tradition and partly by the choice of writers of textbooks. The textbooks play an important role. However, since the late 1980-ties here is no longer any process of approval or even evaluation of textbooks or other learning material at central level. Teachers at the local school evaluate textbooks and decide which one to use in their school.

3.2 Upper secondary school

For the upper secondary level the syllabus is more concrete although there still is no detailed information on the content. The main areas are arithmetic, geometry, trigonometry, theory of probability, statistics, algebra, the theory of functions, differential and integral calculus. Those parts that have already appeared in compulsory school are treated in greater depth at this level. (National Agency for Education, 1995 a and b).

All courses contain revision of earlier material. Mathematics A offers very little new material but consists mainly of revised topics. For Mathematics B – E, revision is not predominant.

In Mathematics A trigonometry is introduced, polynomials and exponential functions are treated and more stress is on geometric propositions. Otherwise the same topics are treated as in compulsory school.

In Mathematics B geometry is developed and proofs are introduced in a more systematic way. Probability and statistics is further developed with different measures of central tendency and of spread, the normal distribution, correlation and causal connection. The complete quadratic equation, linear inequalities and systems of linear equations are treated.

In Mathematics C logarithms are defined and used for problem solving, statistical investigations are done and index series are studied. The derivative is introduced and studied.

In Mathematics D trigonometry is treated in a more complete setting (the unit circle) and graphs of trigonometric functions are studied. Differential and integral calculus is developed. Numerical solution of equations and applications of integrals and derivatives are studied.

In Mathematics E finally differential calculus is pushed further and differential equations and complex numbers are treated and a project work is compulsory.

The courses are studied in the Natural Science programme at approximately the following ages: A-B: age 15-16, C-D: 16-17, E: 17-18.

4 Exemplary topics

The presentation in the following is based on commonly used textbooks and to some extent on national tests.

4.1 Quadratic equations

Quadratic equations are introduced in year 9 (age 15 years). At this level quadratic equations without linear term are treated. Equations with more than one quadratic term that can be reduced to linear equations are also treated.

In Mathematics B (upper secondary, age 16 – 17), the complete quadratic equation is treated. The method using completion of the square is treated before the formula based on that procedure is presented. Pupils then are encouraged to learn the formula by heart. The topic is revised in connection with graphs of second degree polynomials and parabolas later on in Mathematics B.

Quadratic equations are treated in Undvall et al (1997) Z pp 128 ff, in Skoogh et al (1998) F pp 190 ff, Björk et al (1994) pp 182 ff and pp 254 ff.

4.2 Pythagorean theorem

Classical geometry is not treated systematically in compulsory school. Proofs and proving is hardly ever discussed. In upper secondary some proofs are shown, but pupils do not reach an understanding of the concept of proof or of the logical reasoning behind the mathematics that they know.

Most pupils will meet the Pythagorean theorem in lower secondary school. Normally it would be introduced in year 9 (age 15 years), often in connection with quadratic equations. Pupils are trained to compute the third side of the right-angled triangle, given two sides. Many pupils also work with composite exercises based on the theorem. Problem solving involving the Pythagorean theorem is done by more advanced pupils.

In some textbooks an illustration of the theorem is presented by a diagram of three squares, placed alongside the sides of the triangle. Some teachers may (wrongly) suggest to their pupils that this is the ”proof”. Some textbooks give the proof of the theorem in an ”advanced” section, aimed for motivated pupils. The proof given is (normally) one based on comparison of areas inside a square with the side equal to the sum of the smaller legs of the triangle.

Some (probably very few) teachers may choose to introduce the theorem much earlier, in year 4 - 6 or 7, in connection with the study of area of triangles.

In Mathematics A (age 16 years) in upper secondary school, the Pythagorean theorem is revised and a proof based on similarity is presented. The proof is probably not treated in any of the vocational programmes.

The first national test on Mathematics A contained a problem on the Pythagorean theorem, in a non-standard setting.

> *National test, Mathematics A, may 1995: Item 15.*
> The figure shows a rectangle with sides of 1.20 m and 0.50 m. You should cut the triangle along the dotted line and investigate how you could assemble the pieces into a triangle. Find the triangle’s perimeter.
> (There is a figure showing a rectangle with the indicated sides where one of the diagonals is dotted. The figure is not in scale and this information is displayed besides the figure. All the workings should be shown.)

Result: Max: 4 points. Solution rate was 33% overall, 66% in the Natural science programme and 13% in some vocational programmes. 13% of students in the Natural science programme got the maximum result, but only 1% of pupils in the vocational programmes identified two different solutions and got the maximum result. 49% of all pupils reached 2 points, most commonly by computing the length of the diagonal and explaining their method.

The Pythagorean theorem is treated in Undvall et al (1997) Z page 130 ff, in Skoogh et al (1998) F pp 128 ff, in Björk et al (1994) AB pp 147.

4.3 Similarity

In compulsory school similarity is introduced in year 8 (age 14 years) or year 9 (age 15 years). In some textbooks the concept is introduced in connection with scales. Scaling is an important topic that is treated fairly thoroughly. In other textbooks similarity is solely connected with traditional triangle geometry.

The top triangle theorem and the theorem of Thales are presented, but probably most of the pupils will not actually work with the topic and many may not even encounter the theorems.

In mathematics A at upper secondary level (16 years) a more general treatment of similarity is presented. The concept is defined for polygons. The top triangle theorem and Thales´ theorem are proved. However these parts are marked ”advanced topics”, and probably not treated at all in many groups, especially in the vocational programmes.

Similarity seldom appears in national tests for year 9 or Mathematics A except in the form of questions related to scales.

Similarity is treated in Undvall et al (1997), Z pp 138 ff, Skoogh et al (1996) C pp 53 ff, (1996) E pp 93 ff, (1998) F pp 134 ff, in Björk et al (1994) AB pp 138 ff.

4.4 Word problems

Word problems are abundant at all levels, in textbooks and in tests.

4.5 Percentage

Percentage is treated repeatedly throughout the school system. Pupils in year 4 or 5 will meet percentages such as 10, 25 and 50, in connection with early work with fractions. In years 6, 7, 8 and 9 the topic is revised. Every year some new aspects are added and more difficult exercises and problems treated. Percentage greater than 100% is introduced and decimal fractions of whole percents is treated. In year 9 most students will meet the concepts ”promille” (per thousand) and growth factor. The more advanced pupils will learn how to use the growth factor efficiently, while other pupils may solve similar problems (exersises) by some other method.

In upper secondary school the topic is again treated in the Mathematics A course, now adding PPM (parts per million) and index series.

The topic is always strongly connected to applications and sometimes also to problem solving.

Example, textbook, year 7:
On a pair of slacks there is a label: ”May shrink 5%”. The slacks are 80 cm long. How short may they become after washing?
Example from a textbook, year 8:
How many fruits correspond to 100% if 40% are 120 fruits?
Example, textbook, year 9:
How much is 18.75% of 86 400 kg?
Example, textbook mathematics A:
The number of subscriptions of one of newspaper changed one year from 48 000 to 45 120. Compute the relative change in percentages.
National subject test, year 9, spring 1998, part B, item 1:
a) Hassan bought a shirt in the sale that originally cost 300 kr. What did he pay for it on sale? Show your workings. (Illustration is showing a sign with 25%).

b) How does this price tag compare with the reduction on the sign above? Explain your reasoning and show your workings. (Illustration with the old price 500 kr crossed out and the new price 400 kr.)
Solution rate: a) 91%, b) 74%.
National test, mathematics A, spring 1999, item 9, A:
80 000 persons saw the first section of a Tele-Soap-Opera. It was a success and the number of viewers increased by 6% each week.
a) How many viewed the programme after two weeks?
b) With what percentage had the number of viewers increased after five weeks?
c) After how many weeks has the number doubled?
Solution rate: 60% (Natural science programme), 38% (Social science programme), 16% (Child recreation programme)
National test, mathematics A, spring 1995, item 10:
A jacket costs 800 kr. The price is first raised by 8% and then lowered by 6%. What is the price after both these changes? Anna solves the exercise like this: 800*1.08*0.94=812,16. The price will be 812 kr. Lisa solves the exercise like this: 800+800*0.08-800*0,06=816. The price is 816 kr. Anna's solution is correct but Lisa's is wrong. Explain what Lisa is doing wrong.
Solution rate: 52%

Percentage is treated in Undvall et al (1996) X pp 152 ff, Y pp 156 ff, (1997) Z pp 45 ff, in Skoogh et al (1995) B pp 52 ff, (1996) C pp 92 ff, (1997) E pp 118 ff, (1998) F pp 100 ff, Björk et al (1994) pp 50 ff.

4.6 Average value, median and mode value

Descriptive statistics is quite an important topic and it is introduced early. The area is well suited to connect to applications and to integration with other subjects.

In the goals to attain at year 9 it is stated that the pupil should know different measures of location and be able to interpret, analyse and evaluate data given in different forms.

Different types of diagrams are introduced at primary level. Average value is also introduced at primary level and revised at lower secondary level. Median and mode are introduced in year 8 (age 14) and pupils are trained to understand the reasons why one should use one or the other in simple cases.

In Mathematics A at upper secondary level the three concepts are revised, different measures of spread are introduced and more composed problems are treated.

National test Year 9, Spring 1999, part B, item 10:
At a company with 15 employees the average wage was 15 800 kr/month and the median wage was 16 000 kr/ month. When two more people were employed the average wage for the company went up to 16 00 kr/month, despite one of the new employees receiving a wage of less than 15 800 kr/month.
a) Suggest what wage each of the two new employees received.
b) The median wage remained unaltered after the two new employees arrived. Explain why.
National test, Mathematics A, spring 1995, timed part, item 14:
Anna and Johan are rolling dice. The results are shown below.
(An illustration showing two diagrams with Frequency and Number of dots on the axis.)
a) Johan rolls the dice 25 times. How is this made clear in the diagrams?
b) Find the median of Anna's results.
c) Calculate the average points if Anna's and Jonah's results are combined.
Solution rate: a) 83%, b) 35%, c) 40 %. In Natural science Programme: a) 95%, b) 57%, c) 58%.

Measures of location are treated in Undvall et al (1996) Y pp 105 ff, in Skoogh et al (1994) A p 143 and (1995) C pp 136 ff and in Björk et al (1994) AB pp 296 ff.

5 Other subjects of interest for mathematics education around 16 year old

5.1 Regional and local characteristics

There are no general regional differences in the education system. However in some regions schools meet more difficulties when recruiting qualified and/or experienced teachers. There is a shortage of teachers in science and mathematics on a national level. This shortage is much more pronounced in some regions (for instance the northern parts of the country). The lack of qualified teachers in some regions and in some types of communities may influence mathematics teaching considerably and negatively.

After the reform of upper secondary school 1994, most municipalities have established upper secondary schools and in 1999 80% offer upper secondary schools with national programmes (National Agency for Education, 2000). Before the reform pupils from minor municipalities were directed to upper secondary schooling in some neighbouring municipality. The number of upper secondary schools offering a whole range of programmes has increased considerably since 1993. Many of these new schools have experienced difficulties and will encounter more in the future, while recruiting teachers, developing competence and establishing good teaching and learning traditions. However, creativity may flourish under such circumstances and therefore the establishing of many new schools during a short period must not be viewed as mainly a negative process.

Independent schools are more common in the main cities.

Some municipalities make greater use than others of their right to shape the school system within the given frames.

One example of a political will to develop the system is Stockholm where the political majority is liberal. A specific programme for inspecting all schools has been adopted at the political level. The inspection is focused on some of the subjects, among them mathematics. Much stress is put on basic skills for all pupils. At the same time the importance of challenging the most advanced students is recognised. Recently a programme was launched for upgrading teachers at the upper secondary level within their subjects to let a number of them attain a research degree.

5.2 Implementation strategy

Municipalities employ teachers. It is the responsibility of the employer to provide teachers with in-service training. Part of teachers working time is allocated to in-service training and competence development. According to an agreement with the union 15 days every three years is for competence development.

Universities, most often Teacher Education Departments offers programmes (one or several days long) with the aim of informing and up-grading teachers.

When a new curriculum or revised syllabi is introduced, teachers devote some of the training time to the new documents. They discuss and decide upon the implementation at the local level.

The National Agency of Education or the Ministry of Education might commission to some department to organise courses or training programmes in connection to the new curriculum. The municipalities or head of schools will then let the teachers attend these programmes.

In connection to the last reform (1993 - 1994) the National Agency of education decided to let a group of researchers write an survey picturing the role of mathematics in school and in the successive curricula ever since school first became compulsory in Sweden in 1842. This booklet (National Agency of Education, 1997) also contains valuable comments on the new syllabus for compulsory school.

During the period 1986 – 1990 a large programme, directly funded by the ministry, was launched and accomplished for in-service training of teachers at primary level. The topic was mathematics and didactics of mathematics. The immediate reason was the depressing result for Swedish 13 years old pupils compared to other nationalities in the second international study in the eighties, SIMS. This programme was judged to be a successful one and in TIMSS in the ninties. Swedish students had improved their results considerably when compared to other countries.

5.3 Teacher training

Teachers at compulsory school are trained at universities or university colleges. The majority of teachers for grade 7 - 9 have a university degree in each of their subjects (two, three or four subjects, in total three years) plus a diploma of a one-year course in theory and practice of teaching. Since 1989 teacher students for this level are trained in an integrated programme for teaching year 4 - 9, with different specialisation according to different combinations of teaching subjects. For mathematics a minimum of one and a half semester of subject matter studies is compulsory.

Teachers of general subjects at the upper secondary level have a university degree in two or three subjects. They also have a one-year course in theory and practice of teaching. From 1993 the minimum requirements are four and a half-year of study with 2 years for the main subject and 1,5 years for a second subject and one year's pedagogical training. There are a few upper secondary teachers with a Ph.D. degree but almost none in mathematics.

The municipality has the responsibility to offer teachers in-service training. Most often the content is not mathematics, but general topics. The universities with a great volume of teacher training and The National Agency for Education may initiate teacher shorter or longer training programmes.

5.4 Resources available for teachers

The journal Nämnaren, published with four issues every year since the 70-ties, is widely spread and contains articles of direct didactical use for mathematics teachers.

A newly created (1999) resource centre at Gothenburg University is called the National Centre for Mathematics Education. The aim is to provide teachers with information and resources to use in mathematics teaching and learning. Other such resource centres exist in other areas, such as physics, chemistry and technology. They are all part of the governments' programme for recruiting more pupils into science and technology.

5.5 Problems already detected

During the last years an intense debate has been going on about the transition from secondary to tertiary level. The transition from lower to upper secondary level is also an urgent problem. Teachers at the higher level experience that new students or pupils enter with deteriorated knowledge and skills in mathematics compared to earlier years.

Teachers at engineering education initiated the current debate on the transition to university level. The government commissioned two investigations. The results are reported in Johansson (1998) and National Agency for Higher education (1999). A new structure for the upper secondary level was suggested in the last report, and a new branch for mathematics and computer science proposed. This suggestion has been positively received. The debate and process is described in Brandell et al (1999).

5.6 Data on national/regional results

Testing and assessing on a national level has been done since several decades and a lot of data is gathered. Besides results for a random sample of the pupils other data is assembled e g information on the type of school and the type of municipality. Teachers are asked to give their views on the tests by answering inquires.

However no systematic data is gathered with the aim of comparing the development over time. Therefore it is not easily done to investigate development of pupils knowledge as measured by these tests over time.

5.7 Examples of inspiring activities

Every second year since 1980 a great event involving a large number of mathematics teachers has been the Mathematics Biennal. Over 4000 teachers attended the Biennal in

Gothenburg in the year 2000. More than 350 different lectures, seminars and workshops were offered, besides exhibitions and other activities, all related to the teaching of mathematics in year K-12.

Many of the lectures, exhibitions and workshops are devoted to the presentation of untraditional or innovative work done by teachers.

5.8 New teacher education

There have been a number of reports during the last years pointing to weaknesses in the functioning of teacher education. By a decision of the Swedish Parliament in 1998 a committee consisting of politicians from all parties represented in the Parliament has investigated the goals and structure of teacher training. The committee put forward a suggested reform of the teacher training in June 1999. The reform is now discussed and a decision is expected during the year 2000.

The main idea is that all different kinds of teacher education should be unified into one structure. All becoming teachers will have as part of their education three semesters training in common. During this part general subjects will be studied, aiming at competence in the area of schooling and education. Work placement is part of this common section. The academic character of teacher education is reinforced. All teachers graduating in the new system will be eligible for research studies in some area. The programmes for pre-school teachers are prolonged and integrated into the general teacher-training programme.

5.9 New curricula and syllabi from year 2000

New curricula has been decided and will start to apply from the year 2000 - 01. The the reform 1993 - 94 was a major one. Evaluation has pointed to some problems that have evolved during the implementation of the reform and the difficulties detected call for some changes.

For compulsory school only minor changes have been suggested. Still, the tendencies are important. More stress is put on the goals to strive towards compared to the goals to attain. The three major components of learning in mathematics are described as basic skills, conceptual knowledge and problem solving. These three components are more focused than in the previous syllabus. The coupling to everyday mathematics is weakened. Motivation and challenges for students with good potential in mathematics is stressed.

The consequences of the new curriculum may be limited and it is only after some years the impact of the changes can be evaluated.

For upper secondary level the changes are more important. Up to now the Natural science programme has had two branches, the Natural science and the Technical branch. They actually have much in common. A new programme is now introduced namely the Technical programme. The Natural science programme is revised and a new branch is introduced that is called the mathematics - computer - branch. This reform may well turn out to be a major improvement for mathematics at this level. Not since the 1960-ties has there been a specialisation towards mathematics in the Swedish upper secondary school. A new course in discrete mathematics will be developed as compulsory part of this branch. More optional courses may be developed locally or on a national level.

Besides this change of the structure, a revision is made of Mathematics A – E. More points and thus teaching time is allocated to some of the courses, and the proportion of mathematics of the whole teaching time will thus increase. More stress is put on basic algebra, proofs and proving and the use of calculators and computers is more strongly recommended. The content of the courses is only marginally changed.

5.10 Government puts money into in-service teacher training

As a direct consequence of the discussions and investigations into the transition problem (from secondary to tertiary level) the government has decided to fund a mathematics programme. The decision was taken in November 1999. The programme will be directed towards mathematics teachers in school and teachers of mathematics education in the teacher training departments. The content will be mathematics and didactics of mathematics. The National Centre for Mathematics Education will present a plan for this teacher training programme in 2001 and the programme will be carried out during 2001 and 2002 according to the government decision.

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Websites:
National Agency for Education: www.skolverket.se
Swedish Ministry of Education and Science: at the www.gov.se

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Undvall Lennart, Olofsson Karl-Gerhard, Forsberg Svante (1996) *Matematikboken Y röd*, Liber utbildning, Stockholm

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Björk Lars-Eric, Borg Kenneth, Brolin Hans (1994) *Matematik 2000 Kurs AB*, Natur och Kultur, Stockholm

Less traditional and more innovative:

Jacobsson Sven, Wallin Hans, Wiklund Staffan (1995) *Matematik Program N Kurs A+B* Liber utbildning, Stockholm

Skoogh Lennart, Ahlström Ronny, Björlin Jan, Torbjörnsson Lena (1994*) Möte med matte A*, Liber, Stockholm

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